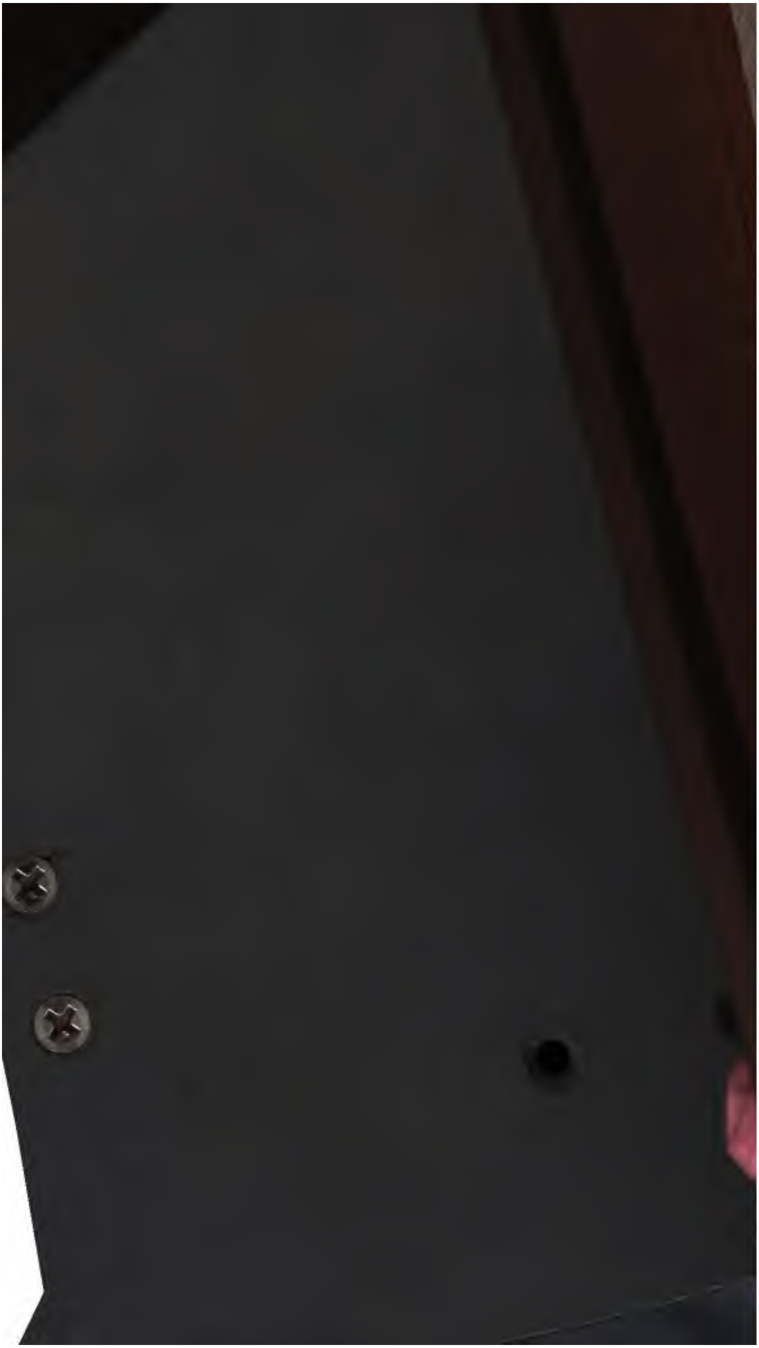

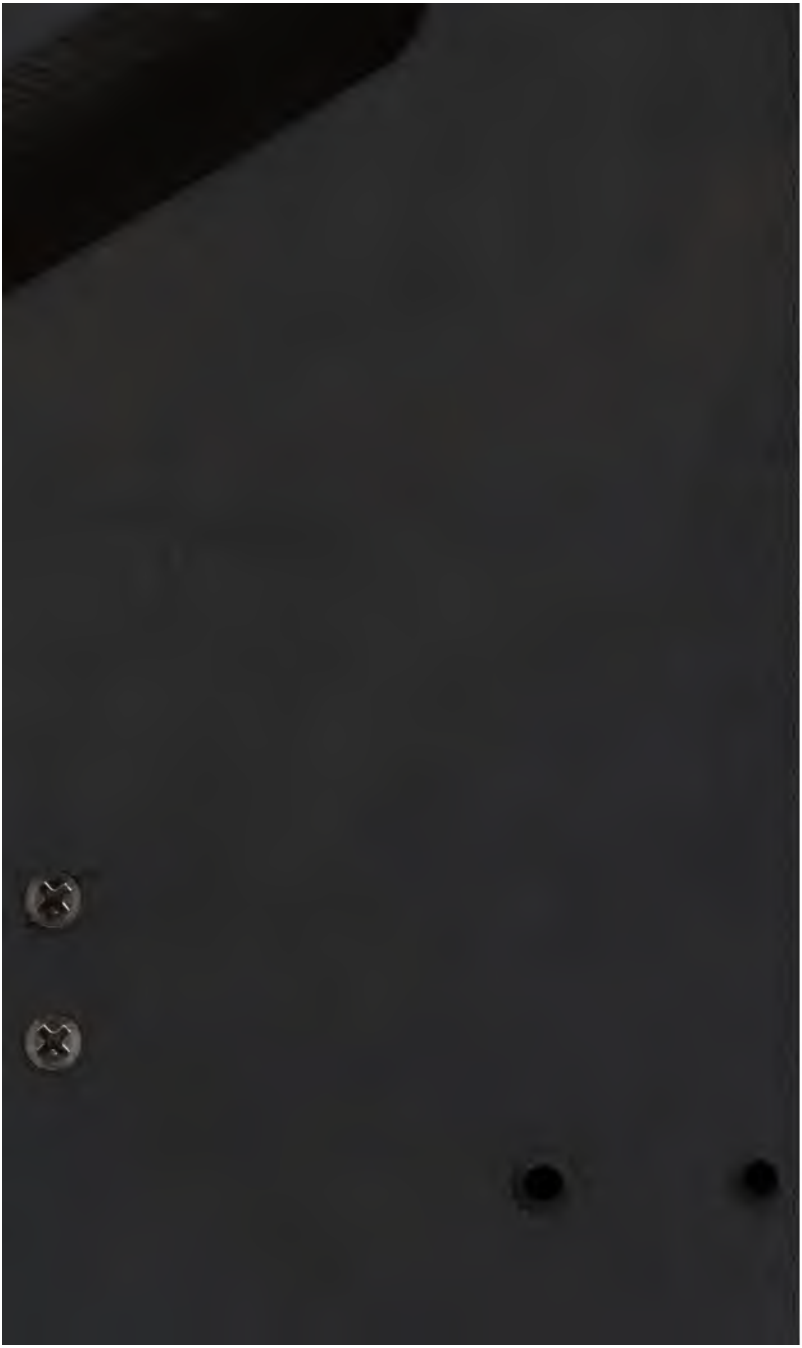

APPENDIX

TO THE

NATURAL ARITHMETIC

FOR TEACHERS' USE

BY

Z. RICHARDS, A. M.

CHICAGO
THE INTERSTATE PUBLISHING COMPANY
BOSTON: [illegible] FRANKLIN STREET

APPENDIX

TO THE

NATURAL ARITHMETIC

FOR TEACHERS' USE

BY

Z. RICHARDS, A. M.

CHICAGO
THE INTERSTATE PUBLISHING COMPANY
BOSTON: 30 FRANKLIN STREET

CONTENTS.

APPENDIX.

The Nature of Arithmetic.

As the plan of THE NATURAL ARITHMETIC is radically new, and may not be fully understood by many teachers, the essential principles of arithmetic, though fully recognized in the book itself, are formulated below for the special benefit of such teachers.

1. To become master of arithmetic, it is essentially necessary, at the beginning, to learn to read and understand the language, or the terms and signs of arithmetic, which represent numbers; though the meaning of number should be thoroughly taught, *orally*, and *objectively at first.*

2. It is essential to bear in mind that there are only *four kinds of numbers* to be used in arithmetical calculations; and *only four methods* of representing and of using them. The four kinds of numbers are:

1. So-called abstract whole numbers, or common units.
2. Decimal parts of units, or decimal numbers.
3. Common fractional units, or vulgar fractions.
4. Combined units of different names and values, or Denominate Numbers.

3. The value of numbers is changed in two ways, — by *increase* and by *diminution.* There are *two ways* of *increasing* numbers, viz. by addition and multiplication, and *two* ways of decreasing numbers, viz. by subtraction and by

division. Addition, Subtraction, Multiplicatiôn, and Division constitute the so-called Fundamental Rules of Arithmetic.

4. Numbers can also be used *logically*, which constitutes Practical Arithmetic, or the adaptation of numbers to the various employments of life.

There are, therefore, some general principles, which must always be recognized in the use of numbers:

1. The language and meaning of numbers must be clearly understood, before using them, and the characters and signs which represent numbers must be learned just as common word-signs are learned.

2. When units of different names are used, if of the same kind, keep in mind how many units of any lower value make one unit of the next higher value.

3. If units of the *same kind*, but of different values, are to be subjected to any of the four operations peculiar to all numbers, they must first be changed to units of the same denominate value, and then used as common abstract numbers; and then, if necessary, changed to units of higher value.

4. Combined units of different values, if they are of the same nature, may be changed to a common unit value, and then be subjected to any of the four operations of addition, subtraction, multiplication, and division, by using them as common units; and then by changing them to units of higher value, if necessary. For illustration, add £$\frac{1}{4}$ to 6s., £$\frac{1}{4}$ = 5s., and 5s. + 6s. = 11s. Also, add three-fourths of a pound English money to three-fourths of a shilling. Solution: £$\frac{3}{4}$ = 15s. = 180d., and $\frac{3}{4}$s. = 9d., then 180d. + 9d. = 189d. = 15s. 9d. These operations are natural, but a shorter and better method is given under Addition of Denominate Numbers.

The foregoing principles underlie every purely arithmetical operation.

NOTE.—Numbers are of the same *kind* or *nature*, when they can be changed to any other name, without changing their value, and they are of the same *name*, when they have the same denomination.

The foregoing principles should be taught practically when the pupils are learning to read numbers; and they have been explained and illustrated in THE NATURAL ARITHMETIC, but every teacher should master them thoroughly, before attempting to teach arithmetic.

First Steps in Teaching Numbers.

Whenever the teacher begins to teach arithmetic, the *first thing* to be done is to teach the idea of number, bearing in mind that neither a *figure* nor the *name* of a number is number. This idea must be taught, for no child can learn it by study. The child may learn to repeat, or to read the *signs* of numbers from one to a thousand, as 1, 2, 3, 4, 5, etc., but he does not, necessarily, know a single number.

1. Teach the idea or meaning of *one*, not abstractly, but objectively, or concretely.

For example:—Hold up *one* finger, requiring fixed attention, then drop it; then hold it up again, and say "*one* finger." Require the pupils to *do* and *say* the same repeatedly, until they can all do it without a mistake.

Then do the same with *one* pencil, one block, or one anything, when a single thing is pointed out.

2. Next raise two fingers, or two pencils, or two books, and teach the idea of two things until the pupils can recognize two things as readily as one thing.

3. Next in order, after being sure that the *number two* is learned, show *three* fingers, or three pencils, or three objects

of any kind, and teach the number three in the same way, comparing it with two and one.

4. In the same manner, teach the numbers four, five, etc., in their order, until the *idea* of number is fixed in the pupil's mind.

Hitherto the pupil is supposed to have heard only the name of the number; but now there can be no reasonable objection to teaching the *visible* representation of all numbers learned; and this should be done carefully upon the blackboard or slate until the *visible* name will call up the idea of number, as well as the *oral* name, or the specific number of objects. This should be the first lesson in the *sight* language of numbers.

Combining Numbers.

1. Hold up again *one* finger, and when the pupils say *one* finger, put another finger by the side of it, and teach them that one finger and one finger make *two* fingers; or that one pencil and one pencil make *two* pencils. Do the same with balls or blocks.

2. Again, hold up *two* fingers, or two pencils, or two balls, and then put one of each kind with each two, and teach the pupils to say, two fingers and one finger make three fingers, or that one finger and two fingers equal two fingers and one finger, etc.

3. Again, hold up *three* fingers, and add one finger, and teach them that three fingers and one finger make four fingers; and so proceed with all the digits, and also with numbers expressed by two or more combined digits.

4. Again, put *two* fingers with *two* fingers, or with *three* or *four*, or *five*, or any number of objects, and teach the pupils to give accurate amounts readily.

5. Again, take *three* objects, or *four* objects, or *five*, or any number of objects, and combine with each of the groups

four, five, or any number of objects, and continue these operations orally, until the pupils can give accurate results as soon as they see the number of objects.

6. Put the figures or signs which represent each of the concrete numbers heretofore used, neatly upon the blackboard, and teach the pupils to give accurate results *at sight*, abstractly, as two and four equal six, etc.

7. As soon as possible, require the pupils to write these exercises; first, by copying them from the board, and second, from dictation.

NOTE. — The exercises given and suggested above must be repeated and varied until the pupils have mastered them; and the teacher should be able to do this without reading them from a voluminous collection in a book. If teachers are obliged simply to imitate authors, the pupils will most likely be mere imitators of teachers.

Subtracting Numbers.

1. Hold up one finger, and when the pupils say one finger, take it away, and teach them that one finger taken away from one finger leaves no finger. Do the same with pencils and other objects.

2. Hold up two fingers, two pencils, etc., then take away one, and teach them that one finger taken from two fingers leaves one finger, etc.

3. Perform the same operation with three, four, and more fingers. Require the pupils to repeat the operations, and read them from the board.

4. Require the pupils, as soon as possible, to write these exercises on slates or paper, first by copying them from the board, and second, from dictation.

The two operations above pointed out may now be combined; *first*, *objectively*, or with the balls of the Numeral frame; and *second*, *abstractly*, or without objects.

These exercises should be followed up persistently, but

not long enough to become wearisome; this will secure *accuracy* and *rapidity* in adding and subtracting.

If the above work is performed thoroughly, in the manner indicated, it will generally be enough for the first year's training in numbers, of children from five or six years of age to seven. Still some teachers may do more; and some children of that age, whose home environments are favorable, may readily comprehend all that precedes.

Test Examples in Adding and Subtracting Digit Numbers.

Require the pupils to give answers orally and in writing.

Lesson 1.		Lesson 2.		Lesson 3.	
1+1=	1−1=	2+2=	2−2=	3+3=	3−3=
2+1=	2−1=	3+2=	3−2=	4+3=	4−3=
3+1=	3−1=	4+2=	4−2=	5+3=	5−3=
4+1=	4−1=	5+2=	5−2=	6+3=	6−3=
5+1=	5−1=	6+2=	6−2=	7+3=	7−3=
6+1=	6−1=	7+2=	7−2=	8+3=	8−3=
7+1=	7−1=	8+2=	8−2=	9+3=	9−3=
8+1=	8−1=	9+2=	9−2=		
9+1=	9−1=				

Lesson 4.		Lesson 5.		Lesson 6.	
4+4=	4−4=	5+5=	5−5=	6+6=	6−6=
5+4=	5−4=	6+5=	6−5=	7+6=	7−6=
6+4=	6−4=	7+5=	7−5=	8+6=	8−6=
7+4=	7−4=	8+5=	8−5=	9+6=	9−6=
8+4=	8−4=	9+5=	9−5=		
9+4=	9−4=				

Lesson 7.		Lesson 8.		Lesson 9.	
7+7=	7−7=	8+8=	8−8=	9+9=	9−9=
8+7=	8−7=	9+8=	9−8=		
9+7=	9−7=				

NOTE.—Only forty-five additions and forty-five subtractions can be performed with the nine digit numbers. See NATURAL ARITHMETIC, pages 20 and 21.

The meaning and use of the signs $+$, $-$, $\times$, $\div$, and $=$ must be carefully taught, thus: $4 + 2 = 6$ should be read four and two equals six, or four plus two equals six; $4 - 2 = 2$ should be read four less two or four minus two equals two; $4 \times 2 = 8$ should be read four times two, or four multiplied by two, equals eight; and $4 \div 2 = 2$ should be read, four divided by two equals two.

Multiplication or Repetition of Numbers.

1. One thing, or a unit, may be repeated by using it twice, three times, or any number of times, thus showing an increase by multiplication. This operation is best taught objectively by the use of the Numeral frame.

2. Two things, or two units, may be repeated any number of times, from once upwards. This can also be shown by the Numeral frame.

For two years the number of repetitions may be limited to ten, or to twelve at most, but the repetition of each of the digit numbers, from one up to ten times, should be thoroughly mastered before taking up higher numbers.

3. Thus *three things*, or *four things*, or any number of things, represented by one of the nine digits, may be repeated any number of times, up to *ten* or *twelve*. At first these operations should be objective, by using the Numeral frame, or objects.

The pupils should be taught to read and copy these numbers from the board, and then repeat them, and write them from dictation, until they have mastered the multiplication table, as given in THE NATURAL ARITHMETIC.

If pupils are made familiar with all the operations suggested in the preceding pages, it will furnish arithmetical work enough for the first two years of school training.

In addition to the mastery of the multiplication and

division tables, as given in THE NATURAL ARITHMETIC, the pupils should be taught to resolve the composite numbers of the tables into all their possible factors. For instance, $12=2\times6$, or 3×4. $16=2\times8$, or 4×4. $20=2\times10$, or 4×5. $24=2\times12$, or 3×8, or 4×6. $28=2\times14$, or 4×7. $30=2\times15$, or 3×10 or 5×6. $32=2\times16$, or 4×8. $36=2\times18$, or 3×12 or 4×9 or 6×6. $34=2\times17$. $35=5\times7$. $38=2\times19$. $39=3\times13$. $40=2\times20$, or 4×10 or 5×8.

NOTE.—Any teacher can extend these and similar examples to almost any degree, with profit to all pupils. Any number of practical business examples can be devised, as illustrations of factoring numbers.

Practical Examples in Factoring.

1. If I give 12 cents for 6 apples, what does one apple cost ?

2. If 3 cents will pay for one apple, how many apples can I buy for 12 cents ?

3. At 7 cents a bunch, how many bunches of grapes can I buy for 14 cents ?

4. If 12 months make a year, how many months in one-fourth of a year ?

5. If a horse travels 7 miles an hour, how many hours will it take him to travel 28 miles ?

6. If sugar is 7 cents a pound, how many pounds can I buy for 35 cents ?

7. If I give 48 cents for 12 oranges, how much will one orange cost ?

8. If I give 3 cents for one orange, how many can I buy for 39 cents ?

9. At 5 cents a pound for rice, how many pounds can I buy for 40 cents ?

10. How many bananas at 3 cents apiece can I buy for 36 cents ?

Division of Numbers.

The foregoing examples practically teach Division; as composite numbers are resolved into factors by finding how many times one number is contained in another; or how many times one number can be subtracted from another.

Thus, $8 \div 2 = 4$; or $8 - 2 - 2 - 2 - 2 = 0$. Here we find 2 is subtracted four times.

In all oral instruction, multiplication and division should be taught together. If the number 4 repeated four times makes 16, then 16 will contain 4 four times: and if $5 \times 5 = 25$, then $25 \div 5 = 5$. Exercises similar to these should be repeated continually, to secure accuracy and rapidity.

Caution. — Do not give young beginners examples containing large numbers or large factors, until they can readily operate with the nine digit numbers, and the composite numbers produced by any two of these digits. Be sure that the multiplication and division tables are thoroughly mastered.

Exercises for the Blackboard and Slate.

Require the pupils to write from dictation.

Lesson 1.	**Lesson 2.**	**Lesson 3.**	**Lesson 4.**
$12 - 2 =$	$\frac{1}{2}$ of $4 =$	$8 + 4 =$	$9 + 3 =$
$12 - \quad = 9$	$\frac{1}{4}$ of $12 =$	$12 - 4 =$	$9 + \quad = 12$
$9 + 3 =$	$\frac{1}{2}$ of $8 =$	$+ 9 = 12$	$8 + 4 =$
$9 + \quad = 12$	$\frac{1}{4}$ of $8 =$	$12 - 9 =$	$4 + \quad = 12$
$2 + 10 =$	$\frac{1}{8}$ of $8 =$	$12 - \quad = 3$	$5 + 7 =$
$2 + \quad = 12$	$\frac{1}{3}$ of $9 =$	$10 + \quad = 12$	$7 + \quad = 12$
$12 - 10 =$	$\frac{1}{5}$ of $10 =$	$+ 6 = 12$	$12 - 5 =$
$12 - \quad = 2$	$\frac{2}{3}$ of $9 =$	$7 + \quad = 12$	$12 - 7 =$

Lesson 5.	Lesson 6.	Lesson 7.	Lesson 8.
$2 \times 7 =$	$\frac{1}{2} \times 14 =$	$9 + \quad = 15$	$\frac{1}{3} \times 15 =$
$14 \div 7 =$	$\frac{1}{2} \times 12 =$	$15 - 6 =$	$\frac{1}{5} \times 15 =$
$14 \div 2 =$	$\frac{1}{3} \times 12 =$	$15 - 9 =$	$\frac{1}{3} \times 12 =$
$6 \times 2 =$	$\frac{1}{4} \times 12 =$	$6 + \quad = 15$	$\frac{1}{6} \times 12 =$
$12 \div 2 =$	$\frac{1}{6} \times 12 =$	$15 \div 3 =$	$\frac{1}{4} \times 12 =$
$12 \div 6 =$	$\frac{1}{3} \times 9 =$	$15 \div 5 =$	$\frac{1}{7} \times 14 =$
$3 \times 4 =$	$\frac{1}{4} \times 8 =$	$5 \times \quad = 15$	$\frac{1}{4} \times 8 =$
$6 \times \quad = 12$	$\frac{1}{5} \times 10 =$	$15 - \quad = 10$	$\frac{1}{3} \times 9 =$
$\times 3 = 12$	$\frac{1}{8} \times 8 =$	$14 - 8 =$	$\frac{1}{7} \times 14 =$

Exercises in Reading Numbers.

We can hardly enforce too strongly the importance of mastering arithmetical language, or the terms which represent the four kinds of numbers.

When the preceding lessons are mastered, however, the pupils ought to be able to read, *at sight*, all the digits, single and combined, up to 1000, at least in Arabic characters; and up to 100 in Roman characters, so far as they represent abstract whole numbers.

They will now readily learn to read and write the various combinations of the digits which represent all kinds of numbers.

They must be taught that annexing a cipher to any digit increases its unit value ten times; thus 1 unit with a cipher annexed becomes ten units, and 9 units with a cipher annexed becomes ninety units, or nine tens. A cipher annexed to 10 will make 100 units, or one hundred.

In the next place, teach the pupils that when any digit figure is annexed to any other digit, or to any combination of digits, the value of such digit, or of the combined digits, is increased ten times, plus the value of the annexed digit.

Thus, if 8 is annexed to 2, the result will be $20+8=28$. Again, if 6 be annexed to 55, the result will be 6 added to ten times $55=550+6=556$. The pupils should be drilled with similar exercises until they can readily read any combination of digits up to millions. This instruction should be given carefully and thoroughly, until the pupils can promptly call any number seen, and write any number dictated.

Reading Decimals.

The language of the next class of numbers to be learned is that of decimal numbers, or of common units divided into *ten equal parts*. The whole number of any name may be divided, or supposed to be divided, into ten equal parts or tenths; and any one of these parts may be divided into ten other equal parts, whose name will be hundredths, and a hundredth may be divided into ten equal parts called thousandths, and so on to millionths.

The meaning of decimals should be taught objectively, by actually dividing a line, a strip of paper, or some object into ten equal parts. The whole line or object will always equal *ten-tenths*, or $\frac{10}{10}$; and one of these equal parts will be one-tenth, or $\frac{1}{10}$, or .1. Two of these parts will be two-tenths, or $\frac{2}{10}$, or .2. Five of these parts will be five-tenths, or $\frac{5}{10}$, or .5. In the same manner, any number of these parts less than ten may be expressed; but the pupil must be taught that ten of these parts always make the whole unit, or one.

The pupil should be made familiar with the fact that tenths always occupy the *first* place to the right of the separator, or period-mark, that hundredths take the *second* place to the right, thousandths the *third* place, ten-thousandths the *fourth* place, hundred thousandths the *fifth* place, millionths the *sixth* place, etc.

1. Read the following examples of whole numbers and decimals:

First, whole numbers, as 100, 101, 116, 127, 147, 157, 169, 179, 188, 195, 201, 212, 223, 235, 246, 1,006, 1,026, 1,038, 1,049, 1,050, 1,062, 1,075, 1,095.

2. Read decimal numbers as follows: .5, .13, .23, .35, .46, .55, .75, .87, .95, 1.05, 1.15, 1.25, .01, 1.01, 2.34, .475, 4.2,625, .23,456, .234,567.

The teacher can add any number of similar examples for reading matter which may be necessary to secure accuracy and rapidity in reading numbers.

3. Change the following *literal* numbers to numbers expressed in Arabic characters, by writing them.

EXAMPLES. — Five tenths, nine tenths, fifteen hundredths, seventy-five hundredths, one hundred and twenty-five thousandths, six hundred and seventy-five thousandths, one thousand and fifteen ten-thousandths, six hundred and twenty-five hundred-thousandths, five millionths, five and five millionths.

The teacher should dictate a sufficient number of similar examples to the pupils, or write them literally upon the blackboard, to familiarize them with reading and writing all kinds of numbers in Arabic characters.

Reading and Simplifying Common Fractions.

When the pupils have learned to read all the fractions given under this head in the main book, they can hardly fail of being able to read any common fraction. But they must be thoroughly impressed that when any unit or single thing is divided into any number of equal parts except ten, hundred, thousand, etc., the parts are called vulgar or common fractions, and have specific names, according to the number

of parts the unit is divided into. Thus, if the unit is divided into two equal parts, these parts are called halves, and one part is called one-half, or $\frac{1}{2}$; if into three equal parts, they are called thirds, and one part is called one-third, or $\frac{1}{3}$; and two parts, two-thirds, or $\frac{2}{3}$; if into four equal parts, they are called fourths, and one part is called one-fourth, or $\frac{1}{4}$; two parts, $\frac{2}{4}$; three parts, $\frac{3}{4}$; and four parts, $\frac{4}{4}$, or one whole unit, or 1.

Illustrate this subject further by dividing the unit into any number of equal parts; as into 20 parts, 25, 50, 75, 125, 625, etc. Show also that the *unit* is always equal to the fraction in which the number of parts taken equals the number of parts into which it is divided: thus $1 = \frac{20}{20}$, or $\frac{25}{25}$, $\frac{50}{50}$, $\frac{75}{75}$, $\frac{625}{625}$, etc.

Reading Denominate Numbers.

The importance of understanding the language of denominate numbers is not generally appreciated. Even if the Metric System should become universal, the principle of denominate numbers will be recognized in meters, decameters, or decimeters; and liters, decaliters, or deciliters, etc., as every kind of number represents some thing which must have a name.

First. The pupils should be taught to read, understand, and repeat from memory, all the common tables of denominate numbers.

Second. Read the following expressions, and as many similar ones improvised by the teacher as may be needed to secure familiarity with the language of denominate numbers.

1. Federal Money: $5, 6d., 8cts., 3mi., or $5.683.

2. English Money: £5, 15s., 8d., 3far.; £25, 6s., 9d., 2far.

3. Avoirdupois Weight: 2T., 15cwt., 3qrs., 21lb., 11oz., 12dr.

4. Apothecaries' Weight: 3lb., 8℥, 7ʒ, 2℈, 15grs.

5. Troy Weight: 5lb., 10oz., 15pwt., 21grs.; 25lb., 11oz., 12pwt., 8grs.

6. Long Measure: 3mi., 5fur., 30rd., 3yd., 2ft., 10in.

7. Square Measure: 2sq. mi., 200A., 3R., 30sq. rd., 25sq. yd., 8sq. ft., 100sq. in.

8. Cubic Measure: 20cu. ft., 1525cu. in., or 3cd., 2cd. ft., 7cu. ft.

9. Dry Measure: 3bu., 3pk., 7qt., 1pt.

10. Liquid Measure: 3tun, 1pi., 1hhd., 40gal., 3qt., 1pt., 3gi.

11. Time Measure: 10cent., 30yr., 8mo., 3wk., 5da., 18hr., 55min., 40sec.

12. Circular Measure: 1cir., 8s., 25°, 50′ or mites, 45″ or motes.

The pupils should learn to read at sight the numbers and signs as abbreviated above, and write them when their names are pronounced.

As all the future work of the pupil in arithmetic will supply continuous examples in reading numbers, the foregoing will furnish sufficient preliminary reading matter in numbers. But the teacher must be sure that all arithmetical language is understood by the pupils.

Test Exercises in Addition.

No part of arithmetic is generally so inefficiently taught, and imperfectly learned, as Addition; while in practical life it is required oftener than any other part. A few more practical examples, therefore, will be given here; but teachers should improvise many other examples.

1. If Maine has 32,000 square miles, New Hampshire 9,000, Massachusetts 7,800, Vermont 9,000, Rhode Island 1,200, and Connecticut 4,700, how many square miles in all New England?

2. If New York has 47,000 square miles, New Jersey 8,300, Pennsylvania 46,000, Delaware 2,000, and Maryland 9,300, how many square miles in these Middle States?

3. The population of Maine in 1880 was 648,936; of New Hampshire, 346,991; of Vermont, 332,286; of Massachusetts, 1,783,085; of Rhode Island, 276,531; and of Connecticut, 622,700; how many inhabitants in New England?

4. The population of New York in 1880 was 5,082,771; of New Jersey, 1,131,116; of Pennsylvania, 4,282,891; of Delaware, 146,608; of Maryland, 934,943; what was the population of these Middle States?

5. What was the population of New England and the Middle States?

6. The four most populous cities in our country in 1880 were New York, Philadelphia, Brooklyn, and Chicago. The population of New York was 1,206,299; Philadelphia, 847,170; Brooklyn, 566,663; and Chicago, 503,185; what was their total population?

7. What was the total population of the 16 cities the population of each of which, in 1880, was between 100,000 and 500,000: viz.: 1. Boston, 362,839; 2. St. Louis, 350,519; 3. Baltimore, 332,313; 4. Cincinnati, 255,139; 5. San Francisco, 233,959; 6. New Orleans, 216,090; 7. Washington, D. C., 160,502; 8. Cleveland, 160,146; 9. Pittsburg, 156,389; 10. Buffalo, 155,134; 11. Newark, 136,508; 12. Louisville, 123,758; 13. Jersey City, 120,722; 14. Detroit, 116,340; 15. Milwaukee, 115,587; 16. Providence, 104,857?

8. How many inhabitants were there, in 1880, in the 20 cities named above ?

9. Find the sum of the following numbers, United States money:

$5,673,911,987.87+	
44,376,013,705.90	$2,581,381.25+
32,673,231,695.25	20,041,121.75
7,736,910,286.16	565,380.40
6,444,642,155.14	25,125.87
44,297,763,429.39	1,202,441.28
26,105,321,266.57	17,182,125.61
9,708,132,873.63	468,350.50
42,231,001,161.86	1,581,345.75
63,497,476,084.03	225,146.81
1,362,004,706.22	14,891,500.50
$	$

10. Footing of United States Revenue Reports.

SAMPLE.

Ports of Entry.	Balance.	Fees.	Salaries.	Fines and Penalties.	Deposits.	Totals.
Passamaquoddy,	$ 13,079.87	$ 4,915.62	$ 500.00	$573.15	$ 342.17	
Portsmouth,		409.76	186.96		370.79	
Boston,	18,204.03	24,197.89		33,000.70	41,352.91	
Providence,	100.93	2,444.45	150.00	11.25	4,497.63	
New Haven,		1,415.03	187.50	18.75	5,028.41	
New York City,	22.67	102,338.74		35,324.33	211,415.35	
Albany,		691.70	600.00		3,889.36	
Newark,		386.90	113.89		46.22	
Philadelphia,	600.62	20,621.02		2,615,42	42,157.63	
Baltimore,	380.19	5,603.23		215.50	20,810.91	
Totals,						$ 598,821.48

DIRECTIONS: — 1. Add each column separately for bottom totals.
2. Add the numbers in the lines oposite the ports of entry for side totals.
3. Add the bottom totals and the side totals separately. Each sum of totals must equal $ 598,821.48.

Examples in Addition of Decimals.

The above examples in Federal Money illustrate decimals as far as hundredths, and if mills are used they are represented by thousandths. All decimal numbers are added like whole numbers, when those numbers of the same name are written directly under each other, observing, always, that the *separator*, or decimal point, is next to the right of units, and also next to the left of tenths.

NOTE. — See the lessons in reading decimals, page 15.

1. Add 21.74, .075, 103.00375, .005495, and 4957.5.

2. Add 3.25lbs., 47.348lbs., 748.4lbs., and 29.32lbs.

3. Add 2.8146, .0938, 8.875, 231.2788, and 4.0087.

4. Add 54.3ft., 7.29ft., 180.0046ft., 187ft., and 3.024ft.

5. Add 3.0102, 11.5008, 73.07, 2.92, and 9.5.

6. One field contains 5.3 acres, a second contains 11.43 acres, a third contains 17.59 acres, and a fourth contains 3.175 acres; how many acres in all of them?

7. In 5 piles of wood there are, respectively, 4.316 cords, 8.23 cords, 11.25 cords, 7.364 cords, and 13.819 cords; how many cords in all the piles?

8. Add five hundred, and nine hundredths; three hundred and seventy-five; twenty thousand and eighty-four; seventy-eight hundred-thousandths; eleven millions, and two thousand two hundred and nine millionths; eleven hundred-millionths; one billion, and one billionth.

9. A farmer has four bins of wheat. In the first there are 86.35bu., in the second 73.125bu., in the third 96.5bu., and in the fourth 74.3bu.; how many bushels in all?

Examples in Addition of Common Fractions.

Before attempting to add common fractions, the pupils must learn the meaning of them, and how to read them. An arithmetical fraction is one or more of the equal parts into which a unit of any kind is divided.

Fractions to be added, subtracted, or compared must be, (1) of the *same kind or nature*, (2) of the same name, and, (3) of a simple form.

First. They are of the *same kind or nature* when they can be reduced to the same name, as $\frac{1}{2}$lb. and $\frac{1}{2}$oz. Troy can be reduced to drams; viz. $\frac{1}{2}$lb. = 96 drams, and $\frac{1}{2}$oz. = 8 drams; or, $\frac{1}{4}$lb. and $\frac{1}{5}$lb. = $\frac{5}{20}$lb. and $\frac{4}{20}$lb.

Second. They are of the *same name* when they represent equal parts of the same thing, as $\frac{1}{4}$ and $\frac{3}{4}$, and $\frac{3}{8}$ and $\frac{7}{8}$.

Third. They have a *simple form* when they have a simple whole number for both numerator and denominator, as $\frac{4}{5}$ and $\frac{8}{5}$. When the numerator equals or exceeds the denominator, the fraction is called an *improper fraction.*

Common fractions are expressed in four forms; viz., (1) the simple fraction, as $\frac{3}{4}$ or $\frac{5}{4}$; (2) the compound fraction, as $\frac{1}{2}\times\frac{2}{3}$ or $\frac{1}{3}\times\frac{2}{5}$; (3) the complex fraction, as $\frac{\frac{1}{2}\times\frac{2}{3}}{4}$ or $\frac{4}{\frac{1}{3}\times\frac{1}{4}}$ or $\frac{\frac{1}{3}}{\frac{1}{4}}$; (4) the mixed number, as $5\frac{1}{2}$ or $6\frac{3}{8}$.

Any one of these fractions becomes a *denominate fraction* when the name of the parts must be expressed, as $\frac{1}{2}$lb. or $\frac{1}{4}$oz. or \$$\frac{1}{5}$, etc.

To prepare fractions for arithmetical operations, the pupils must be taught how, first, to simplify them, and second, to reduce them to the same name. When thus prepared, they can be used like simple whole numbers, if the name is always kept in mind.

First. To simplify a fraction, it must be so changed in

form as to have a simple whole number for the numerator and also for the denominator.

Fractions may be thus changed by performing the operations required by the signs in their natural order.

1. Simplify	compound	frac.	$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$	a simple fraction.
2. "	divided	"	$\frac{1}{2} \div \frac{3}{4} = \frac{2}{4} \div \frac{3}{4} = \frac{2}{3}$	"
3. "	combined	"	$\frac{1}{2} + \frac{3}{4} = \frac{2}{4} + \frac{3}{4} = \frac{5}{4}$	"
4. "	subtracted	"	$\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4}$	"
5. "	complex	"	$\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{1}{2} \div \frac{1}{3} = \frac{3}{6} \div \frac{2}{6} = \frac{3}{2}$	"
6. "	mixed	"	$4\frac{1}{2} = \frac{9}{2}$	"

Second. To reduce fractions of *different names* to equivalent fractions having the same name.

1. Find, by inspection, the smallest number which can be measured by each of the denominators without a remainder, for a new denominator.

2. For the new numerator, take such a part of the new denominator, thus found, as the given fraction is of unity.

1. Examples in Simplifying Fractions.

1. Simplify			$\frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6} = \frac{1}{6}$.
2. "	and add		$(\frac{1}{2} \div \frac{2}{3}) + (\frac{3}{4} \div \frac{1}{5}) + (\frac{4}{5} \div \frac{1}{6}) = 9\frac{3}{10}$.
3. "	"	"	$\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} = ?$
4. "	"	"	$(\frac{1}{2} - \frac{1}{3}) + (\frac{3}{4} - \frac{1}{5}) + (\frac{4}{5} - \frac{1}{6}) = ?$
5. "	"	"	$\frac{\frac{1}{2}}{\frac{1}{3}} + \frac{4}{\frac{3}{4}} + \frac{\frac{2}{3}}{5} = 6\frac{29}{30}$
6. "	"	"	$4\frac{1}{2} + 5\frac{1}{3} + 6\frac{1}{4} = ?$

2. Examples in Reducing Fractions to the Same Name, and then Adding them.

Each of the last six examples furnishes specimens both of simplifying fractions and of reducing them to the same name.

1. Simplify and add $\frac{2}{3}+\frac{\frac{1}{4}\times\frac{3}{5}}{3}+\frac{\frac{2}{3}\div\frac{1}{6}}{4}=\frac{2}{3}+\frac{1}{20}+1=1\frac{43}{60}$.

2. " " " $\frac{1}{3}+2\frac{5}{7}+8\frac{7}{9}=\frac{21}{63}+2\frac{45}{63}+8\frac{49}{63}=11\frac{52}{63}$.

3. " " " $270\frac{3}{4}+650\frac{3}{20}+5000\frac{1}{4}+53\frac{4}{5}+1\frac{1}{20}=\,?$

4. " " " $2\frac{2}{3}+3\frac{3}{4}+4\frac{4}{5}+5\frac{5}{6}+6\frac{6}{7}=\,?$

5. " " " $\left\{\frac{\frac{2}{3}\times\frac{3}{4}}{\frac{1}{2}}+\frac{\frac{2}{5}\times\frac{5}{6}}{\frac{2}{9}\times 4\frac{1}{2}}\right\}\div\left\{\frac{\frac{2}{3}\times\frac{3}{4}}{\frac{1}{2}}-\frac{\frac{2}{5}\times\frac{5}{6}}{\frac{2}{9}\times 4\frac{1}{2}}\right\}=\,?$

6. " " " $\left\{\frac{2+\frac{4}{5}}{2-\frac{4}{5}}-\frac{2-\frac{4}{5}}{2+\frac{4}{5}}\right\}\div\frac{2-\frac{2}{2\frac{2}{3}}}{3}=\,?$

NOTE 1.—All fractions can be reduced to lower terms when their numerators and denominators can each be exactly divided by the same number, which does not change the value of the fractions.

NOTE 2.—It will rarely be necessary to find the least common multiple of two or more numbers in any other way than by *inspection* or trial; but, if it should be necessary, use the Rule given in THE NATURAL ARITHMETIC, page 30.

NOTE 3.—The addition of decimals, essentially the same as whole numbers, has been sufficiently illustrated heretofore.

Denominate Fractions.

Denominate fractions are such as always have the name of the parts expressed in abbreviations; thus, £$\frac{1}{2}$ and £$\frac{1}{3}$;

or, $\frac{3}{4}$s., and $\frac{1}{6}$s. To add, subtract, and compare these fractions, they must be prepared like common fractions.

1. Simplify and add $£\frac{1}{2}+£\frac{1}{3}=£\frac{3}{6}+£\frac{2}{6}=£\frac{5}{6}=$?

2. $£\frac{1}{4}+\frac{1}{6}$s. $+\frac{1}{4}$d. $=240$f. $+8$f. $+1$f. $=249$f. $=5$s., 2d., 1f.

3. $\frac{1}{4}$yr. $+\frac{3}{4}$mo. (30 days) $+\frac{3}{4}$da. $+\frac{3}{4}$h. $+\frac{3}{4}$min. = ?

4. $\dfrac{2.3475\text{ton}+4.6875\text{cwt.}+(.5\times 3\text{qrs.}-1.4\text{lb.})}{1.05\text{ton.}}=2\frac{10}{21}.$

Some Essential Methods for Changing Fractions.

FIRST. *All common fractions can be reduced to an equivalent decimal form, when simplified.*

RULE. — 1. *Actually divide the numerator by the denominator, if it equals or exceeds the denominator; if not, reduce it to tenths by annexing a cipher, and then divide.*

2. *If there is a remainder, annex another cipher for hundredths and divide; and so continue to do until a sufficient degree of exactness is reached.*

1. Reduce $\frac{1}{2}$ to an equivalent decimal form.

ILLUSTRATION. $\frac{1}{2}\times 10=5$, but the decimal form requires the decimal point immediately before the 5, for tenths, thus .5, which is equivalent to five-tenths, or $\frac{5}{10}$.

2. Reduce $\frac{4}{5}$ to a decimal, thus, 5)4.0(.8 = *Ans.*

3. " $\frac{3}{4}$, also $\frac{27}{40}$. *Ans.* .75 and .675.

4. " $\dfrac{\frac{3}{4}}{3}$ to a decimal: simplify this complex fraction, thus $\frac{3}{4}\div 3=\frac{1}{4}=.25.$

5. Reduce $\dfrac{6}{1+\frac{1}{2}}$ to a decimal. *Ans.* 4.

6. " $\frac{3}{4}\times\frac{17}{20}$ to a decimal.

SECOND. *All decimal fractions can be reduced to equivalent common fractions by writing the decimal form as a numerator, over the number which expresses the name of the decimal, for a denominator.*

7. Reduce .04 to a common fraction.

ILLUSTRATION. $.04 = \frac{4}{100} = \frac{1}{25}$.

8. Reduce .45 to a common fraction.

9. " " $.45\frac{3}{8}$ to a common fraction.

ILLUS. $\frac{45\frac{3}{8}}{100}$: first simplify, and $\frac{45\frac{3}{8}}{100} = \frac{363}{8} \div 100 = \frac{363}{800} =$ *Ans.*

10. Reduce .00049 to a common fraction.

11. " .675, also .000003.

Condensed View of Common Fractions.

1. The Meaning of Fractions.

1. The kind. 2. The name. 3. The form.

First. They are of the *same kind* when they represent a certain part of the same thing; as $\frac{3}{5}$lb. and $\frac{2}{3}$oz., sugar.

Second. They are of the *same name* when they represent equal parts of the same value and of the same thing; as $\frac{3}{5}$lb. and $\frac{7}{5}$lb.

Third. They have a *simple form* when the numerators and denominators are simple whole numbers; as $\frac{6}{7}$, $\frac{8}{7}$.

Fourth. If fractions are to be added, subtracted, or compared, they must be of the *same kind*, the *same name*, and *simple form*.

Fifth. The *forms* of fractions are (1) simple, as $\frac{1}{5}$, $\frac{6}{5}$; (2) compound, as $\frac{1}{2} \times \frac{5}{7}$, or $4\frac{1}{2} \times \frac{3}{5}$; (3) complex, as $\frac{\frac{1}{2} \times \frac{2}{3}}{4\frac{1}{2}}$; (4) mixed, as $4\frac{1}{2}$, $5\frac{2}{3}$.

2. How Fractions are Used.

1. *Fractions of the same name and of simple form.*

1. Add $\frac{5}{4}+\frac{3}{4}=\frac{8}{4}=2$.

2. Subtract $\frac{5}{4}-\frac{3}{4}=\frac{2}{4}=\frac{1}{2}$.

3. Multiply $\frac{5}{4}\times\frac{3}{4}=\frac{5}{4}\times 3\div 4=\frac{15}{16}$.

4. Divide $\frac{5}{4}\div\frac{3}{4}=\frac{5}{3}=1\frac{2}{3}$.

2. *Fractions of different names, but of same kind and form.*

1. Add $\frac{3}{4}+\frac{4}{5}=\frac{15}{20}+\frac{16}{20}=\frac{31}{20}=1\frac{11}{20}$.

2. Subtract $\frac{3}{4}-\frac{2}{5}=\frac{15}{20}-\frac{8}{20}=\frac{7}{20}$.

3. Multiply $\frac{3}{4}\times\frac{2}{5}=\frac{6}{20}$.

4. Divide $\frac{3}{4}\div\frac{2}{5}=\frac{15}{20}+\frac{8}{20}=\frac{15}{8}=1\frac{7}{8}$.

3. *Fractions of different forms, but of the same kind.*

1. Add $(\frac{1}{2}\times\frac{2}{3})+\frac{3}{5}=\frac{1}{3}+\frac{3}{5}=\frac{5}{15}+\frac{9}{15}=\frac{14}{15}$.

2. Subtract $(\frac{1}{2}\times\frac{2}{3})-\frac{1}{5}=\frac{1}{3}-\frac{1}{5}=\frac{5}{15}-\frac{3}{15}=\frac{2}{15}$.

3. Multiply $(\frac{1}{2}\times\frac{2}{3})\times\frac{3}{5}=\frac{1}{5}\times\frac{3}{5}=\frac{1}{3}$.

4. Divide $(\frac{1}{2}\times\frac{2}{3})\div\frac{3}{5}=\frac{1}{3}\div\frac{3}{5}=\frac{5}{15}\div\frac{9}{15}=\frac{5}{9}$.

4. *All denominate numbers can be reduced or changed to equivalent decimals of a higher or of any given denomination.*

12. Reduce £5, 5s. to the decimal of a £.

ILLUSTRATION. 5s. $=\frac{1}{20}$ or $\frac{1}{4}$ of one pound; and £$\frac{1}{4}$ = £.25; to be annexed to the £5 = £5.25.

But a simple and ready method of changing denominate numbers to decimals of the highest denomination is to write the terms (beginning with the lowest) directly under each other in their order, and divide, first the lowest, by the

number of its units required to make a unit of the next higher name, and annex the quotient to the next higher name; and so continue until the highest name is reached.

13. Reduce £2, 6s., 8d., 2far. to the decimal of a £.

ILLUSTRATION.

4	2f.
12	8.5d.
20	$6.708\frac{1}{3}$s.
	£$2.3354\frac{1}{6}$

Also 2lb., 8oz., 2pwt.,

20	2.0
12	8.100
	2.675lb.

14. Reduce 7 miles, 7fur., 1rd. to the decimal of a mile.

15. Reduce 4bu., 3pk., 5qt., 1pt. to the decimal of a bush.

16. Reduce £2, 6s., 8d., 3far. to the decimal of a shilling.

Ans. 46.7291 + s.

17. Reduce 15lb., 10oz. to the decimal of a cwt.

Ans. .15625cwt.

5. *The value of a decimal denominate number can be found in units of other denominations by* (1) *multiplying the given decimal by the number of tabular units of the next lower name required to make a unit of the denomination used.* (2) *Point off, as in decimals, and the figures on the left of the separator will be the units required.* (3) *Continue this process with the decimal part of each product till the lowest unit value is found.*

18. Find the denominate value of .890625 of a bushel.

ILLUSTRATION.

.890625
4pks.
3,562500
8qts.
4,500000
2pts.
1,000000

	pk.	qt.	pt.
Ans.	3	4	1

19. Find the denominate value of .86 of a cwt.

20. What is the value of £.3375?

21. What is the value of .3375ton?

22. What is the value of .575acre?

23. What is the value of $\frac{3}{8}\times .86$cwt.?

ILLUSTRATION. $\frac{3}{8}\times .86\text{cwt.} = .3225\text{cwt.}$ Reduce as before.

5. *Denominate numbers may be changed to equivalent fractions of any given name or denomination.*

First, write the given denominate number or numbers for a numerator, and one unit of the required denomination for a denominator.

Second, reduce, if necessary, the numerator and denominator to units of the same value, and then reduce the fraction to its lowest terms.

24. Reduce 2s., 6d. to the fraction of a £.

ILLUSTRATION. $\frac{2\text{s. }6\text{d.}}{£1} = \frac{30\text{d.}}{240\text{d.}} = £\frac{1}{8}$ *Ans.*

25. Reduce 3gal., 2qt. to the fraction of a hogshead.

ILLUSTRATION. $\frac{3\text{gal. }2\text{qt.}}{1\text{hhd.}} = \frac{14\text{qt.}}{252\text{qt.}} = \frac{1}{18}$hhd. *Ans.*

26. What part of a cwt. is .9lb.? *Ans.* $\frac{9}{1000}$cwt.

27. What part of a guinea is 12s., 9d., $1\frac{1}{3}$far.? *Ans.* $\frac{115}{189}$guinea.

28. What part of an acre is 2R., 32sq. rd., 8sq. yd.? *Ans.* $\frac{849}{1210}$acre.

6. *We can find the value of a common denominate fraction, of any higher denomination, in units of lower denominations, as follows:—*

RULE. — *Take such a part of the tabular number of the next lower denomination as is indicated by the given fraction,*

and, if there is a fractional remainder, reduce it in the same manner to the next lower denomination; and so on to the lowest name.

29. What is $\frac{4}{5}$lb. Troy in denominate numbers?

ILLUSTRATION. $\frac{4}{5} \times 12$oz. $= 9\frac{3}{5}$oz.; $\frac{3}{5}$oz. $= \frac{3}{5} \times 20$pwt. $= 12$pwt.
Ans. 9oz., 12pwt.

30. What is $\frac{7}{8}$tun of wine? *Ans.* 1pi., 1hhd., 1bbl.

31. What is $\frac{6}{7}$ of 3 days, in lower denominations?
Ans. 2da., 13h., 42min., $51\frac{3}{7}$sec.

32. What is $\frac{1}{2} \times \frac{3}{4} \times 6\frac{2}{3}$bush.? *Ans.* 2bush., 2pk.

33. What is $\frac{7}{9}$mile? *Ans.* 6fur., 8rd., 4yd., 2ft., 8in.

34. What is $\frac{15}{16}$ of a common year?
Ans. 11mo., 7da., 12h.

Exercises in Subtraction.

The subject of subtraction has been sufficiently discussed and illustrated in THE NATURAL ARITHMETIC.

1. Examples in Whole Numbers.

1. Find the difference between 1 and 1000.

2. The difference between 6521 and 3857.

3. The difference between two thousand and nine, and ten thousand and ninety-six.

4. What number added to eight hundred and seventy thousand, five hundred and eighty-nine, will amount to twenty millions, eight thousand, six hundred and seventy?

2. Examples in Decimals.

5. From thirty-five thousand take thirty-five thousandths.

6. What is the difference between one and five-tenths, and three thousand, seven hundred, and eighty-five ten thousandths?

7. What is the difference between 213.5 and 1.8125?

NOTE.—To make pupils familiar and quick in expressing decimals when dictated orally, the following directions are given:—

1. The teacher should repeat orally the decimal number, and require the pupils to write the figures dictated just as if they represented whole numbers.

2. Make the pupils determine the number of decimal places at the right of the separator, as indicated by the meaning of the last figure named in the decimal number.

3. Place the separator immediately before the figures, with ciphers prefixed, if necessary, to make up the number of decimal places.

For example: one hundred and twenty-five ten-thousandths, all will write first 125. Now as the last figure, 5, represents ten-thousandths, it must have the fourth place from the separator, with a cipher prefixed to the 125, thus, .0125. Dictate similar but varied examples, to secure familiarity.

3. Examples in Common Fractions.

To subtract common fractions, first simplify and prepare them as in addition, and then subtract the numerator of the subtrahend from the numerator of the minuend, and place the remainder over the common denominator.

8. From $\frac{5}{7}$ take $\frac{4}{9}$.

ILLUSTRATION. Get the fractions to the same name by *inspection*. The least common denominator is 63. $\frac{5}{7}=\frac{45}{63}$; $\frac{4}{9}=\frac{28}{63}$; and $\frac{45}{63}-\frac{28}{63}=\frac{17}{63}$, *Ans.*

9. From $\frac{3}{7}\times 1\frac{1}{2}\times 7$ take $\frac{5}{4}\times\frac{3}{8}$.

SOLUTION. $\frac{3}{7}\times\frac{3}{2}\times\frac{7}{1}=\frac{9}{2}-\frac{15}{32}=4\frac{16}{32}-\frac{15}{32}=4\frac{1}{32}$, *Ans.*

10. From $\frac{4}{11}\times\frac{22}{18}\times\frac{1}{3}$ take $\frac{3}{12}\times\frac{3}{7}$. *Ans.* $\frac{31}{756}$.

11. From $4\frac{1}{6}$ take $2\frac{1}{7}$.

12. From $16\frac{5}{8}$ take $5\frac{8}{9}$.

13. From a cask containing $41\frac{1}{2}$gal., $17\frac{5}{8}$gal. are drawn; by how much does the part left exceed the part taken?

14. From $7\frac{1}{2}-(3\frac{2}{3}+\frac{3}{5})$ take $1\frac{1}{6}\times\frac{2}{5}$. *Ans.* $2\frac{23}{30}$.

15. From $7\frac{1}{2}-3\frac{2}{3}+\frac{3}{5}$ take $1\frac{1}{6}\times\frac{2}{5}$. *Ans.* $3\frac{29}{30}$.

4. Subtraction of Denominate Numbers.

Denominate numbers are subtracted from denominate numbers upon the same general principle as whole numbers, by (1) writing the numbers of the subtrahend under those of the same name in the minuend, and (2) by keeping in mind the *number* of equal parts or units of any name which make one unit in the next higher name, and using that *number* as *ten* is used in common whole numbers.

16. Take £3, 12s., 8d., from £6, 8s., 5d.
Ans. £2, 15s., 9d.

17. From £25, 17s., $4\frac{1}{2}$d. take £19, 19s., $9\frac{3}{4}$d.
Ans. £5, 17s., $6\frac{3}{4}$d.

18. From £3 take 3s. *Ans.* £2, 17s.

19. From 2lb. (Troy) take 20grs.
Ans. 1lb., 11oz., 19pwt., 4gr.

20. From $3\frac{1}{2}$lb. (Troy) take $\frac{1}{6}$oz.
Ans. 3lb., 5oz., 16pwt., 16gr.

21. From $\frac{4}{7}\frac{9}{2}$lb. take $\frac{7}{16}$℥ (Apoth.)
Ans. 7℥, 4ʒ, 2℈, 10qr.

22. From $\frac{6}{7}$cwt. take .2125qr.
Ans. 3qr., $1.0817\frac{6}{7}$lb.

NOTE.—To find the difference of time between any two dates: for common business purposes (1) write the larger number, representing the year, (2) the number of the month, and (3) the number of the day of the month, or the whole of the later date, for the minuend.

2. Under these numbers write the corresponding numbers representing the earlier date.

3. Subtract as required under denominate numbers.

23. How long is it since the American Revolution began, April 19, 1775, to present date?

24. How long since the Declaration of Independence?

25. A note is dated March 15, 1882, and paid July 1, 1885; how long did it run on interest?

Exercises in Multiplication.

1. Simple or Whole Numbers.

In multiplying, the multiplier is considered an abstract number, and the product will be of the same name as the multiplicand. The multiplier and multiplicand are often called factors.

An abstract number is units, or tens, hundreds, etc., used without reference to things, as 1, 2, 3, etc.; but when it represents things, as 1lb., 2s., 3d., or $5, it is called a concrete or denominate number.

Units multiplied by units produce units; units multiplied by tens, or tens by units, produce tens; units multiplied by hundreds, or hundreds by units, produce hundreds, etc.; but tens multiplied by tens produce hundreds, and tens multiplied by hundreds, or hundreds by tens, produce thousands, etc.

These principles should be made familiar to all pupils, so that they will have very little use for the common rule for multiplication given in the first part of THE NATURAL ARITHMETIC.

ILLUSTRATION. 5 units $\times$ 5 units $=$ 25 units, or 2 tens and 5 units; again, 5 tens $\times$ 5 tens $=$ 50 $\times$ 50 or 25 hundreds, or 2 thousands and 5 hundreds; again, 5 tens $\times$ 5 hundreds $=$ 25 thousands, or 2 tens of thousands and 5 thousands, etc.

1. Multiply four hundred sixty-two thousand, six hundred and nine by the number itself.

2. Multiply forty-nine millions, forty thousand, six hundred and ninety-seven by nine millions, forty thousand, seven hundred and nine.

3. There are 20 pieces of cloth, each containing 37 yards, and 49 other pieces each containing 75 yards; how many yards in all the pieces?

4. A farmer bought a farm containing 10 fields; three of these fields contain 9 acres each, 3 other fields contain 12 acres each, and the remaining fields contain 15 acres each; how many acres in the whole farm, and how much did the farm cost at $18 per acre?

5. A regiment of men contains ten companies; each company, 8 platoons: and each platoon, 34 men; how many men in the regiment?

6. The product of two numbers is three hundred and twenty-three thousand, seven hundred and ninety-six. One of them is thrice the difference between ten thousand and one, and nine thousand, nine hundred and ninety-seven. Find the number.

7. *Bill of Parcels.*

Find the amount of the following bill:

WASHINGTON, April 1, 1886.

ANTHONY HYDE bought of H. R. MILES:

27 Bags of Coffee at $14 per bag	$	
18 Chests of Tea at $25 per chest . . .		
75 Barrels of Shad at 9\frac{1}{5}$ per bbl.		
87 " " Mackerel at 8\frac{1}{4}$ per bbl. . .		
67 Cheeses at 2\frac{1}{8}$ each		
59 Hogsheads of Molasses at $29 per hhd. . .		
	$	

Received Payment,

H. R. MILES,

per A. S. RICHARDS.

2. Multiplication of Decimals.

The multiplication of decimals involves three operations: —

1. The multiplication of a decimal number by a whole number or by a common fraction.
2. The multiplication of a whole number or a common fraction by a decimal number.
3. The multiplication of a decimal number by a decimal number.

The general rule for multiplying decimals is the same as that for multiplying whole numbers.

(*a*) If a decimal be multiplied by a simple abstract number, the name of the product will be the same as that of the decimal multiplicand.

(*b*) If a simple abstract number be multiplied by a decimal, the product will be such a part of the abstract number as the multiplying decimal is a part of unity.

NOTE. — As multiplication implies increase, strictly speaking, a number cannot be multiplied by unity or by a number less than unity.

(*c*) If a decimal be multiplied by a decimal, the result will be such a part of the decimal multiplicand as the multiplying decimal is of unity.

8. (*a*) $.5 \times 5 = \frac{25}{10} = 2.5$. Also $.5 \times \frac{2}{3} = \frac{5}{10} \times \frac{2}{3} = \frac{10}{30} = \frac{1}{3}$.

9. (*b*) $5 \times .5 = 2.5$. Also $\frac{2}{3} \times .5 = \frac{1.0}{3.0} = \frac{1}{3}$.

10. (*c*) $.5 \times .5 = \frac{5}{10} \times \frac{5}{10} = \frac{25}{100} = .25$.

From the above results we see that the decimal products have just as many decimal figures, or places, as there are decimal places in both factors, and we have then the following

RULE. — *Multiply as in whole numbers, and point off on the right of the product as many figures for decimals as there are decimal places in both factors.*

11. Multiply three and forty-nine thousandths by twelve thousandths.

ILLUSTRATION.

$$\begin{array}{r} 3.049 \\ .012 \\ \hline 36588 \\ 0000 \\ \hline .036588 \end{array}$$

Ans. Thirty-six thousand, five hundred eighty-eight millionths.

ANOTHER ILLUSTRATION. $\frac{3.049}{1000} \times \frac{12}{1000} = \frac{36588}{1,000,000} = .036588$

12. Multiply \$341.45 by seven thousandths.

Ans. \$2.39+.

13. Multiply .075 by 100.

14. What will 6.29 weeks' board amount to at \$2.79 per week?

15. Find the continued product of $2.03 \times 203. \times .00203 \times 20300$.

16. A man travelled $7\frac{3}{4}$ hours by rail, at the rate of 22.75 miles an hour; $9\frac{3}{5}$ hours by stage, at the rate of 6.75 miles an hour; and 11.75 hours on foot, at the rate of 4.62 miles an hour; how far did he travel in all?

3. Multiplication of Common Fractions.

1. To multiply a fraction by an abstract whole number, multiply the numerator by the whole number, or, if possible, divide the denominator, and reduce the result to the simplest form.

2. To multiply an abstract whole number by a fraction, multiply the whole number by the numerator, and divide the product by the denominator; or, when possible, divide by the denominator first and then multiply the quotient by the numerator.

N. B.—See note on page 35, on multiplying by a number less than unity.

3. To multiply a fraction by one or more other fractions, reduce all fractions of the same nature to simple fractions, and then multiply the numerators together for a new numerator and the denominators together for a new denominator.

NOTE. — We have heretofore learned that any common fraction can be multiplied by any number, —*first,* by multiplying the numerator, or, *second,* by dividing the denominator by that number.

Two or more fractions to be multiplied are called a compound fraction, and the process may be shortened by cancelling equal factors in the numerator and the denominator.

17. Multiply $\frac{17}{19}$ by 5. *Ans.* $\frac{17 \times 5}{19} = \frac{85}{19} = 4\frac{9}{19}$.

18. Multiply $\frac{175}{45}$ by 9. *Ans.* $\frac{175}{45 \div 9} = \frac{175}{5} = 35$.

19. Multiply $2\frac{3}{5}$ by 8. $2\frac{3}{5} = \frac{13 \times 8}{5} = \frac{104}{5} = 20\frac{4}{5}$.

NOTE. — Generally, if the mixed number is small, it is better to reduce it to an improper fraction, and then multiply as in simple common fractions, as illustrated above. But if the multiplicand is a large mixed number, the fractional part may be multiplied first, and reduced if necessary, and then added to the product of the whole number by the multiplier.

20. Multiply $175\frac{3}{5}$ by 9.

ILLUSTRATION. $\frac{3}{5} \times 9 = \frac{27}{5} = 5\frac{2}{5}$. $5\frac{2}{5} + 175 \times 9 = 5\frac{2}{5} + 1575 = 1580\frac{2}{5}$.

Again, a mixed number may be multiplied by a mixed number by simplifying each factor, and then multiplying the numerators together and dividing the product by the product of the denominators.

21. Multiply $128\frac{3}{4}$ by $118\frac{2}{3}$. $128\frac{3}{4} = \frac{515}{4}$ and $118\frac{2}{3} = \frac{356}{3}$. $\frac{515}{4} \times \frac{356}{3} = \frac{183340}{12} = 15278\frac{1}{3}$. Or, $128\frac{3}{4} \times \frac{2}{3} = 85\frac{5}{6}$, and $128\frac{3}{4} \times 118 = 15192\frac{1}{2}$. $15192\frac{1}{2} + 85\frac{5}{6} = 15278\frac{1}{3}$. *Ans.*

22. Multiply $5\frac{1}{2}$ by $5\frac{1}{2}$. *Ans.* $30\frac{1}{4}$.

23. What is the cost of $15\frac{3}{4}$ cords of wood at $\$3\frac{6}{7}$ per cord? *Ans.* $\$60\frac{3}{7}$.

4. Multiplication of Denominate Numbers.

In the multiplication of denominate numbers, the factors must always be used as abstract numbers, but the product will represent the same name as the multiplicand.

The multiplication of denominate numbers does not introduce any new principle. The figures representing different names are multiplied, and then changed, if necessary, to higher numbers by using the appropriate tabular numbers. But the tables for denominate numbers must be made as familiar as the multiplication table.

24. Multiply 8cwt., 3qrs., 1lb., 9oz., by 7.
Ans. 61cwt., 1qr., 10lbs., 15oz.

25. Multiply 13yds., $2\frac{1}{3}$ft., by 9. *Ans.* 124yds.

26. Multiply 5cwt., $2\frac{1}{3}$qrs., by 24.
Ans. 6T., 14cwt., 1lb., 15oz.

27. Multiply £3, $14\frac{5}{6}$s., by 33. *Ans.* £123, 9s., 6d.

28. Multiply $39\frac{1}{4}$yds. by $6\frac{1}{2}$. *Ans.* 255yds., $4\frac{1}{2}$in.

Exercises in Division.

1. Division of Whole Numbers.

In performing division, the divisor and dividend must be used as abstract numbers; but the quotient will represent the same name as the dividend.

The dividend is always equal to the product of the divisor and quotient.

General Rule, applicable to the division of the four kinds of numbers: —

1. Be sure to make the divisor and dividend of the same name.

2. Find how many times the number of units in the divisor is contained in the dividend, or in that part of it representing the highest name.

3. If the divisor is larger than the number representing the highest name, or if there is a remainder after division, reduce the number or remainder to the next lower name, and then divide the reduced numbers; and so continue until the whole number is divided, writing the several quotient factors *in the same order* as the several partial dividends. If there is a final undivided remainder, write it over the divisor as a common fraction.

1. Divide 725,436 by 9, and by 99.

2. Divide 695,748,546 by 5,468.

3. The remainder is 73, the divisor 46, and the quotient 124. What is the dividend?

4. Divide $4032 between A, B, C, and D, so that, as often as A has $2, B shall have $3, C shall have $4, and D shall have $5. How many dollars will each have?

2. Division of Decimal Numbers.

1. *To divide a whole number by a decimal number.*

5. Divide 25 by .5; or twenty-five by five-tenths. Notice the general rule.

ILLUSTRATION. Reduce 25 to tenths by annexing one cipher. 25=250 tenths, which divided by 5 tenths equals 50 units. *Ans.*

6. Divide 675 by .75.

SOLUTION. $675.00 \div .75 = 900$.

7. Divide 678 by 1.25. *Ans.* 542.4.

8. A man divided his estate, worth $10,000, as follows: giving .125 of it to his daughter, three times as much to his son, and four times as much to his wife. How much did he give to each, and had he any property left?

2. *To divide a decimal by a whole number.*

9. Divide .75 by 75.

ILLUSTRATION. Reduce the 75 to hundredths, and then $.75 \div 75.00 = \frac{1}{100}$, or .01. *Ans.*

10. Divide .575 by 1150. *Ans.* .0005.

11. Divide .005 by 25. *Ans.* .0002.

3. *To divide mixed decimals by mixed decimals, and by decimals.*

12. Divide 112.1184 by 9.16.

ILLUSTRATION. $1121184 \div 91600$. *Ans.* 12.24.

13. Divide 95.5 by .25. *Ans.* 382.

4. *To divide decimals by decimals.*

As heretofore directed, reduce the dividend and divisor to the same name, and then divide as in whole numbers.

14. Divide 46.103 by 2.14. *Ans.* $46.103 \div 2.140$.

15. If 75.3 cords of wood cost $640.05, how much will one cord cost? *Ans.* $640.05 ÷ 75.30.

16. Divide 7.8 by .125. *Ans.* $7.800 \div .125 = 62.4$.

NOTE. — If the foregoing principles and illustrations are made familiar, operations in decimals will be as simple as in whole numbers.

3. Division of Common Fractions.

1. To divide a fraction by a whole number, divide the numerator or multiply the denominator by the whole number. Reason: Dividing the numerator lessens the number of parts taken, while the size of the parts remains unchanged; and multiplying the denominator lessens the size of the parts while the number of parts is unchanged.

17. Divide $\frac{18}{37}$ by 9.

ILLUSTRATION. $\frac{18 \div 9}{37} = \frac{2}{37}$; or $\frac{18}{37 \times 9} = \frac{18}{333} = \frac{2}{37}$.

18. $\frac{15}{11} \div 6 = \frac{15}{66} = \frac{5}{22}$. In this case, 6 is not a factor of 15 and must be multiplied by the denominator 11.

2. To divide a whole number by a fraction, reduce the whole number to parts of the same name as the fractional divisior, and then divide this number of parts by the numerator of the dividing fraction.

19. Divide 15 by $\frac{3}{5}$.

ILLUSTRATION. $15 = \frac{75}{5}$. $\frac{75}{5} \div \frac{3}{5} = \frac{75}{3} = 25$.

20. If I pay $\$\frac{3}{7}$ for a bushel of apples, how many bushels can I buy for $9? *Ans.* $9 \div \frac{3}{7} = \frac{63}{7} \div \frac{3}{7} = \frac{63}{3} = 21$.

3. To divide a fraction by a fraction. Reduce both fractions to equivalent fractions of the same name, and then divide the numerator of the dividend by the numerator of the divisor. Or, by a short and common process, invert the terms of the divisor and then proceed as in multiplying fractions.

21. Divide $\frac{4}{5}$ by $\frac{1}{3}$.

ILLUSTRATION. $\frac{4}{5} = \frac{12}{15}$ and $\frac{1}{3} = \frac{5}{15}$, then $\frac{12}{15} \div \frac{5}{15} = \frac{12}{5} = 2\frac{2}{5}$; or $\frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$.

22. Divide $\frac{9}{10}$ by $\frac{15}{16} = \frac{72}{80} \div \frac{75}{80} = \frac{72}{75} = \frac{24}{25}$.

23. Divide $\frac{8}{9}$ by $\frac{13}{18}$. *Ans.* $\frac{8}{9} = \frac{16}{18}$. $\frac{16}{18} \div \frac{13}{18} = \frac{16}{13} = 1\frac{3}{13}$.

24. Divide $\frac{5}{6} \times \frac{13}{25}$ by $\frac{8}{19} \times \frac{19}{21}$.

Ans. $\frac{13}{30} \div \frac{8}{21} = \frac{91}{210} \div \frac{80}{210} = \frac{91}{80} = 1\frac{11}{80}$.

4. To divide a mixed number by a whole number, reduce the mixed number to a simple, improper fraction, and divide as under No. 1, above.

25. Divide $3\frac{1}{2}$ by 10. $\$3\frac{1}{2} = \$\frac{7}{2}$. $\frac{7}{2} \div 10 = \$\frac{7}{20} = 35$ cts.

26. If 12 men consume $6\frac{2}{5}$lbs. of meat in a day, how much does one man consume? *Ans.* $\frac{8}{15}$lb.

5. To divide a whole number by a mixed number, write

the whole number as a fraction with 1 for the denominator: then simplify the mixed divisor and proceed as in No. 3, above.

27. Divide [illegible] by [illegible].

Ans. [illegible]

28. Divide [illegible] by [illegible]. *Ans.* [illegible]

4. To divide a mixed number by a mixed number, reduce each term to an improper fraction, and divide as in No. 3, above.

29. Divide [illegible] by [illegible].

Ans. [illegible]

30. Divide [illegible] by [illegible]. *Ans.* [illegible]

31. If [illegible] ton of hay is worth [illegible], what is a ton worth?

Ans. [illegible], or [illegible].

It is often necessary to find the cost of unity, when the whole quantity and whole cost are given.

RULE.—*Divide the whole cost by the number which represents the whole quantity.* (See Ex. 31.)

32. Paid [illegible] for [illegible] acres of land: how much did I pay per acre? *Ans.* [illegible]

33. If I pay [illegible] for [illegible] turkeys, how much do I pay for one turkey? *Ans.* [illegible]

34. What is the cost of one yard of ribbon, if 3.15yds. cost [illegible] of a dollar? *Ans.* [illegible]

35. Find an equivalent simple fraction equal to (1) [illegible], (2) [illegible], (3) [illegible], (4) [illegible].

Answers. (1) [illegible]. (2) 10. (3) [illegible]. (4) [illegible].

4. Division of Denominate Numbers.

The general Rule for the Division of Denominate Numbers is the same, in principle, as that given for division of whole numbers, on page 38.

First: Divide the number representing the highest denomination, if it contains more units than the divisor; otherwise reduce it, or any remainder, to the next lower name, and add to it the number (if any) of the next lower name; and, Second: Divide the sum, placing the quotient in the same order of units. Continue this process until the lowest name is divided.

36. Divide £33, 7s., 8d., 2far. by 8.

ILLUSTRATION. £33 ÷ 8 = £4, with £1 undivided, which equals 20s. This added to 7s. = 27s. 27s. ÷ 8 = 3s., with 3s. undivided, equaling 36d. 36d. + 8d. = 44d. 44d ÷ 8 = 5d. with 4d. undivided, equaling 16far. 16far. + 2far. = 18far. 18far. ÷ 8 = $2\frac{1}{4}$far.

Ans. £4, 3s., 5d., $2\frac{1}{4}$far.

37. Divide £972, 15s., 10d., 2far. by 132, or the factors 11 × 12. *Ans.* £7, 7s., 4d., $2\frac{1}{2}\frac{8}{2}$far.

38. Divide £45, 15s., 3d. by £2, 3s., 7d.

ILLUSTRATION. £45, 15s., 3d = 10983d.; and £2, 3s., 7d. = 523d.

Ans. 10983d ÷ 523 = 21.

39. Divide £9, 9s., $7\frac{5}{16}$d. by $3\frac{3}{4}$.

Ans. £2, 10s., $6\frac{3}{4}$d.

Exercises in Longitude and Time.

It often becomes necessary to find the difference in time caused by difference in longitude, also, to find the difference of longitude indicated by the difference in time, between any two places.

The process is simple and easy when the tables heretofore given are understood and made familiar. To find the differ-

ence in longitude, keep in mind that we begin the reckoning of longitude at Greenwich, England, and that the difference of time is caused by the revolution of the earth from west to east, and that, as the sun is relatively stationary, the meridian, or line of noon, is always moving westward, in relation to us, so that all places east of a given meridian have *later* time, and all places west of said meridian have *earlier* time, than is indicated at such meridian. Hence, to find the difference of time between any two places on the earth, we must first find the difference in longitude, in *degrees*, *mites*, and *motes* (or degrees, minutes, and seconds, as usually given).

The longitude and time-table is based upon the following facts:

1. One revolution of the earth on its axis makes any given line of longitude pass through a circle, or 360°, in 24 hours of time.

2. One twenty-fourth of a revolution (or one hour) will pass through $\frac{1}{24} \times 360° = 15°$.

3. One degree of motion will require $\frac{1}{15}$ of 1 hour, or 60 minutes $= 4$ minutes of time.

4. One mite (or minute) of motion will require $\frac{1}{60} \times 4$ minutes of time, or 4 seconds of time.

5. One mote (or second) of motion will require $\frac{1}{60} \times 4$ seconds $= \frac{4}{60} = \frac{1}{15}$ of one second of time.

We have then the following table. (See NATURAL ARITHMETIC.)

15° in longitude make 1 hour in time.
15′ (mites or minutes) make 1 minute in time.
15″ (motes or seconds) make 1 second in time.

or, 1° degree of longitude makes 4 minutes of time.
1′ mite (or minute) makes 4 seconds of time.
1″ mote (or second) makes $\frac{1}{15}$ second of time.

1. If 38°, 38′, 45″ is the difference in longitude between two places, what is the difference of time?

ILLUSTRATION. 15) 38°, 38′, 45″

2h., 34m., 35sec. = *Ans.*

2. If the difference of time between two places is 3h., 54m., 28sec., what is the difference of longitude?

Ans. 58°, 37′.

3. If the difference of time, as in the last question, is 77°, 3′ W., between Washington and some place west of it, at noon, what is the longitude of that place and what is the time?

Ans. Long. 135°, 40′, and the time 8h., 5m., 32sec. A.M.

4. Columbus, O., is 83°, 3′ W.; Baltimore 76°, 37′ W. When it is 4 P.M. at Baltimore, what time is it at Columbus?

Ans. 3h., 34m., 16sec.

5. When it is 9 o'clock at Washington, it is 7m., 4sec. past 8 o'clock at St. Louis. What is the difference of longitude? *Ans.* 13°, 14′.

6. Copenhagen is in Long. 12°, 34′, 57″ E., and Mobile is 8°, 11′ W. When it is 10 P.M. at Mobile, what time is it at Copenhagen? *Ans.* 4h., 43m., $3\frac{4}{5}$sec. A.M.

7. When it is midnight at Paris, 2°, 20′, 22″ E. L., what time is it in Chicago, 87°, 35′ W. L.?

Ans. $18\frac{8}{15}$sec. after 6 P.M.

Test Examples in the Metric System.

See the tables and explanations in THE NATURAL ARITHMETIC.

To be an expert in the use of the Metric System is not at present essential for all pupils in this country, and will not be until this system becomes our national system; and

even then, the tables will be needed only to enable us to translate the old denominate numbers into metric numbers.

The nature of the system should be taught, however, until pupils can see that it is essentially the same as our common decimal system, and until they have learned the new nomenclature, or system of naming numbers. A few examples are given here to show how the tables are used in converting denominate numbers into their equivalent metric numbers, and how metric numbers are converted into denominate numbers.

A *meter* is $\frac{1}{10,000,000}$ of the distance from the equator to the North pole measured on a meridian; and, by the law of the United States, it is fixed at 39.37 inches, in Long Measure. The other measures are the *are*, the *stère*, the *liter*, and the *gram*.

The multiple divisions of the various units are named by prefixing to them the Greek numerals deka (10), hecto (100), kilo (1,000), and myra (10,000).

The sub-multiples, or lower denominations, are named by prefixing to the names of the primary units the Latin numerals, deci ($\frac{1}{10}$), centi ($\frac{1}{100}$), mille ($\frac{1}{1000}$). These names correspond to *whole numbers* and *decimals*.

Units of the common system may be readily changed to units of the metric system by the aid of the following table.

1 inch = 2.54 centimeters.	1 cubic inch = 16.39 cubic centim.
1 foot = 30.48 centimeters.	1 cubic foot = 28.320 cubic centi.
1 yard = .9144 meter.	1 cubic yard = .7646 cubic meter.
1 rod = 5.029 meters.	1 cord = 3.625 stères.
1 mile = 1.6093 kilometers.	1 fluid ounce = 2.958 centiliters.
1 sq. inch = 6.4528 sq. centimeters.	1 gallon = 3.786 liters.
1 sq. foot = 929 sq. centimeters.	1 bushel = .3524 hectoliter.
1 sq. yard = .8361 sq. meters.	1 Troy grain = 64.8 milligrams.
1 sq. rod = 25.29 centiares.	1 Troy pound = .373 kilometers.
1 acre = 40.47 ares.	1 Avoir. pound = .4536 kilos.
1 sq. mile = 259 hectares.	1 ton = .907 tonneau.

1. How many inches in 7 meters? *Ans.* 275.59in.

2. How many meters in 605 inches?

Ans. 15.367in.

3. How many miles in 12 kilometers?

Ans. 7.4564mi.

4. How many kilometers in 15 miles?

Ans. 24.1402kil.

5. How many quarts (liquid) in 41 liters?

Ans. 43.3247qts.

6. How many liters in 31½ gallons?

Ans. 119.2391l.

7. How many ounces in 711 grams?

Ans. 27.077oz.

8. How many cords in 15 stères? *Ans.* 4.1385c.

9. Reduce 17½ stères of wood to cords.

Ans. 4.8283c.

10. How many yards in 93.75 meters?

Ans. $93.75 \div .9144 = 102.5259$yds.

11. How many square yards in 32.49 square meters?

Ans. $32.49 \div .8361 = 38.258$square yds.

12. How many square meters in a lot, 100 feet long by 25 feet wide? *Ans.* 232.25 square m.

13. The length of a rectangle is 8.7 meters, and its breadth is 7.8 meters, how many square yards does it contain?

ILLUSTRATION. $7.8 \times 8.7 = 67.86$ square meters $\div .8261 = 81.1606$ square yards.

Percent and Percentage.

1. Test Examples and Illustrations.

The pupils should read and study carefully what is contained in THE NATURAL ARITHMETIC on this subject. Percentage is really the application of the principles of decimal numbers to the business of life, and is, therefore, a part of the metric system. It makes the *unit* of arithmetical reckoning *one hundredth*, and requires that every object of such reckoning shall be divided, or supposed to be divided, into one hundred equal parts.

Percentage is applied chiefly to the various uses of money, or to representative values. We buy, sell, borrow and loan, give and receive these representatives. Whenever there is any consideration in the exchange of values, the rules of percentage require such consideration to be expressed by one or more of the one hundred equal parts used. Our national currency is decimal; but if any other currency not decimal is used, some decimal form must be adopted in applying the rules of percentage. All denominate expressions of number can be changed to a decimal form.

Per cent means by the hundred, or rate in hundredths, or a certain number of hundredths.

Percentage is the amount realized by taking a certain per cent of every hundred, or of any number.

1. To find the percentage of any number, multiply the rate per cent by the number.

1. At 6%, what is the percentage on $250?

ILLUSTRATION. 6%, or $.06 \times 250 = \$15.00$. *Ans.*

NOTE.—There are two final results to be sought in working out percentage questions:

1. To ascertain the percentage of a number for one or more periods of time.

2. To find what per cent one number is of another.

The first operation relates to all kinds of interest, discount, commission, profit and loss, insurance, stocks, bonds, brokerage, banking, and exchange.

The second relates to the relation of numbers by comparison.

2. What is $2\frac{1}{2}\%$ of \$650?

3. What is $4\frac{1}{2}\%$ of 875?

4. Find $5\frac{3}{4}\%$ of 2704 miles.

5. Find $\frac{1}{2}\%$ of 1000 oxen.

6. Find $66\frac{2}{3}\%$ of 420 acres.

ILLUSTRATION. $66\frac{2}{3}\% = \frac{2}{3}\%$, and $\frac{2}{3} \times 420 =$ *Ans.*

7. Find $\frac{7}{8}\%$ of \$225.60. *Ans.* \$1.974.

8. A man has a capital of \$12,500. He puts 15% of it in state stocks, $33\frac{1}{3}\%$ in R. R. stocks, and 25% in bonds and mortgages. What per cent has he left? and what its value? and the value of each investment?

9. A man bought a house on the 3d of March for £1,526, 10s., and agreed to pay for it, with interest at $4\frac{5}{8}\%$, on the first day of October. What sum did he have to pay?

Ans. £1,565, 5s., $9\frac{1}{2}$d, nearly.

10. A merchant spent \$1,650 for dry-goods as follows: 34% of his money for calicos, 25% for silks, and the rest for broadcloths. How many dollars did he spend for each class of goods?

Ans. \$412.5 + \$561. and \$676.50 = \$1650.

To find what per cent one number is of another, (1) make the demanding number the numerator, or dividend, and the other number the denominator, or divisor; (2) annex two ciphers (or multiply by 100) to the numerator and divide by the denominator.

1. Three is what % of four?

ILLUSTRATION. Here 3 is the demanding number and numerator, and 4 is the denominator, thus $\frac{3}{4}$: and this fraction is equal to $\frac{3}{4} \times \frac{100}{100} = \frac{300}{400} = .75$, or 75%.

2. 4 is what % of 3? *Ans.* $\frac{4}{3}$, or $1.33\frac{1}{3}\%$.

3. $\frac{1}{2}$ is what % of $\frac{1}{2}$? *Ans.* $\frac{1}{2} \div \frac{1}{2} = .5 \div .5 = 100\%$.

4. 5 is what % of $\frac{1}{2}$? *Ans.* $\dfrac{5 \times 100}{\frac{1}{2}} = \dfrac{500}{\frac{1}{2}} = 1000\%$.

5. $\frac{1}{2}$ is what % of 5? *Ans.* $\frac{1}{2} \div 5$, or $\frac{1}{2} \div \frac{10}{2} = \frac{1}{10} = 10\%$.

6. $\frac{1}{2}$ is what % of $3\frac{1}{2}$? *Ans.* $\frac{1}{7} = 14\frac{2}{7}\%$.

7. .05 is what % of .5? *Ans.* $\frac{.05}{.50} + \frac{1}{10} = 10\%$.

8. There are two schools, the first of which has 50 pupils, and the second 200 pupils. What per cent of the larger school is the smaller! *Ans.* 25%.

9. 50 is $37\frac{1}{2}\%$ of what number? *Ans.* $\dfrac{50}{37\frac{1}{2}}$ or $133\frac{1}{3}$.

10. $\frac{2}{3}$ is 50% of what number? *Ans.* $\frac{2}{3} \div \frac{50}{100} = 1\frac{1}{3}$.

11. What is 25% of $37\frac{1}{2}$? *Ans.* $.25 \times 37.5 = 9.375$.

NOTE.—If we examine the principles of percentage with a view to their application to common business, we shall find:

First, that *percentage* answers to profit or loss on cost or investment.

Second, that *per cent* is the rate or price for unity.

Third, that *amount* answers to cost or investment, plus percentage or *gain*, or minus the loss.

The terms and language used on this subject, as well as other subjects, must be thoroughly understood.

2. Examples in Insurance.

Insurance is a guaranty to make good any loss by fire or any other casualty. The policy is the written contract, and the premium is the sum paid for assuming the risk. The operations are based upon the rules of percentage.

12. A merchant wishes to insure a vessel and its cargo, at sea, valued at $28,800. What is the premium at $1\frac{3}{4}\%$?

ILLUSTRATION. $28,800 × .01$\frac{3}{4}\%$ = *Ans.* $504.

13. Find the insurance premium on a property worth $48,000, at the rate of $1\frac{1}{4}\%$. *Ans.* $600.

14. The stock of a manufacturing company was insured for $96,000, at the following rates: $\frac{1}{2}$ the stock at $\frac{3}{4}\%$; $\frac{3}{4}$ of the remainder at $\frac{7}{8}\%$, and the rest at $\frac{5}{8}\%$. What was the whole amount of insurance paid? *Ans.* $750.

15. A man insures his house, valued at $10,000, at $\frac{5}{8}\%$; his barn for $1,800, at $\frac{7}{8}\%$; and his furniture valued at $3,000, at $1\frac{1}{4}\%$. What premium does he pay?

Ans. $90.75.

16. Shipped 5,000 bbls. of flour, worth $10.50 a bbl., and paid $2,887.50 insurance. What was the rate?

Ans. $2,887.50 ÷ $5,000 × 10\frac{1}{2}$ = $5\frac{1}{2}\%$.

17. What is the premium on $\frac{3}{4}$ of a vessel worth $27.500, at $3\frac{5}{8}\%$, and $\frac{3}{8}$ of the cargo, worth $126.875, at 2%?

Ans. $2,223.75.

3. Examples in Commission.

Commission is the same as percentage, and it is paid to a person as a commission merchant, for buying and selling merchandise, and to a broker for buying and selling stocks, bonds, etc. The *consignor* delivers his *consignment* to the *consignee*. In the account of sales, the amount due the consignor after deducting commission and all expenses, is called *net proceeds.*

18. A real estate broker sells a house for $23,750, at a commission of $1\frac{1}{8}\%$. What must he pay to the principal?

Ans. $23,750 less $\frac{9}{8}\%$ × 23,750 = $23,482.81$\frac{1}{4}$.

19. A commission merchant receives $3,825 to invest in flour, at a commission of 2%. What is his commission?

ILLUSTRATION. $\$3,825 \div 1.02 =$ amount spent for flour, which, subtracted from money sent, equals the commission, $= \$75$.

20. A broker receives $7,000 to buy cotton at 8 cents a pound, his commission being $1\frac{3}{4}$%. How much does he buy?

Ans. 85,995lbs.

21. An auctioneer sold 150hhds. of sugar, each weighing 1,150lbs., at $7 a hundred, on a commission of $1\frac{1}{4}$%. What was his commission, and how much did he pay to the owner?

Ans. Com. = $149.08, and paid to owner $11,925.926.

4. Examples in Profit and Loss.

Gains and losses are generally reckoned at so much per cent, on property invested, whether in money, stocks, merchandise, etc. Apply the rule of percentage.

22. A merchant buys goods to the amount of $15,625, and sells them at an advance of 20%. What does he gain?

Ans. $3,125.

23. A merchant bought 640 yards of cotton cloth at $6\frac{1}{4}$ cents per yard, and sold the same at $7\frac{1}{2}$ cents per yard; how much did he gain, and what was the gain per cent?

Ans. His gain was $8, at 20%.

24. A grocer buys eggs at 25cts. per doz., and sells them at 20cts. per doz. What is the loss per cent?

Ans. 5%.

25. A man bought a house for $7,625, and sold it for $8,845. What per cent did he gain? *Ans.* 16%.

26. A merchant buys broadcloth at $4.50 per yard. How must he sell it per yard so as to gain 15%?

Ans. $\$4.50 \times 1.15 = \$5.17\frac{1}{2}$.

27. In the last problem, how must he sell his cloth so as to lose 15%? *Ans.* $\$4.50 \times .85 = \$3.82\frac{1}{2}$.

28. A bought 75 barrels of flour at $11 a barrel, but in selling it he lost 20%. How much did he sell it for per bbl.? and how much did he receive?

Ans. $8.80 per lb., and $660.

29. A merchant bought 240yds. silk at 75cts. a yard, and sold it so as to gain 25%. How much did he gain?

Ans. $45.

30. A merchant sells a quantity of flour for $1,848, by which he makes a profit of 12%. How much did the flour cost him? *Ans.* $1,650.

31. I sold a horse for $240 and lost 20%. For what should I have sold him to have gained 10%?

Ans. $330.

32. Sold goods at a loss of 8% on their cost, and received $8,280. What did they cost? *Ans.* $9,000.

33. A man bought 250 bbls. of flour at $\$5\frac{1}{2}$ per bbl., and sold it for $1,875. What per cent did he gain or lose?

Ans. $36\frac{4}{11}\%$.

34. My sales on dry-goods were 25% of my sales on notions, and my gain on notions was 20%. If the loss on dry-goods was 20%, what was the cost of the dry-goods and the notions, the entire sales being $1,200?

Ans. Notions, $800; dry-goods, $300.

ILLUSTRATION. Cost of notions 100% + gain, 20% = 120%. Sales of dry-goods $= \frac{1}{4}$ of 120% = 30%. Sales of both articles = 150% = $1200. Sales of notions $= \frac{120}{150} = \frac{4}{5} \times 1200 = 960$. Sales of dry-goods $= \frac{30}{150} = \frac{1}{5} \times \$1200 = \$240$. Cost of notions $= \frac{5}{6} \times 960 = \$800$, and the cost of dry-goods $= \frac{5}{4} \times \$240 = \$300$.

35. Purchased a house and lot for $4,375. The cost of the house was $133\frac{1}{3}\%$ of the cost of the lot. Sold the

house at 60% profit and the lot at 40% profit. How much did I gain, and for what price did I sell each?

Ans. House, \$4,000; lot, \$2,625; gain, \$2,250.

5. Examples in Taxes.

When property of any kind is taxed, 1st, a certain value is fixed upon it; and, 2d, the amount of the tax to be collected is also determined, and such other conditions as may be necessary. 3d, The rate per cent of tax on one dollar is found by dividing the amount of tax to be assessed by the amount of property to be taxed. 4th, Each person's inventory is to be multiplied by the rate on \$1, to find each person's tax.

Generally there are two kinds of property to be taxed, called personal and real.

Personal property is *movable*, such as furniture, money, bonds, cattle, and the like. Real property is fixed, or immovable, such as houses, lands, etc.

Sometimes there is what is called a *poll tax*, or a certain sum to be paid by certain persons, without regard to property.

The government imposes various kinds of taxes on goods imported, and on some articles manufactured in our own country. A *license* is a kind of tax. Generally the tax is a certain per cent, and the amount is found as percentage is found.

36. I have property assessed at \$36,000, and I pay a tax of $1\frac{1}{8}$ mills on a dollar, for $\frac{2}{3}$ of the value; and there are 3 polls, at 75 cents each. What is my entire tax?

ILLUSTRATION. $\frac{2}{3} \times \$36{,}000 \times .001\frac{1}{8}$ mills $+ 3 \times .75 = \$29.25$.

37. The valuation of the assessed property of a village was \$96,000. Suppose the sum of \$6,360 is needed for the

building of a schoolhouse, what will be the rate of taxation on the property? *Ans.* $6\frac{1}{8}$ mills on a dollar.

38. In a school district a tax of \$375 is levied for the support of schools. What is A's tax on a valuation of \$8,000, the entire valuation of the district being \$120,000? *Ans.* \$25.

6. To find the interest for years and months at any given per cent, or for any number of periods or parts of periods.

RULE. — *Multiply the rate per cent by the number of years or periods, plus the fractional part of periods.*

39. What is the interest of \$5,256.50 at 6% for 2 years, 8 months?

Ans. $\$5,256.50 \times 6\% \times 2\frac{2}{3}$ years $= \$841.04$.

40. What is the interest of \$1,525 at 7%, for 4 years and 6 months? *Ans.* $\$480.37\frac{1}{2}$.

41. What is the interest of \$625.50, at 5% per annum, payable semi-annually, for 2 years, 10 months?

ILLUSTRATION. 2 yrs., 10 mos. $= 5\frac{2}{3} \times 6$ mos.; therefore, $\frac{5}{2}\% \times 5\frac{2}{3} \times \$625.50 =$ *Ans.* $\$88.61\frac{1}{4}$.

42. What is the amount of \$562.50, on interest for 3 years and 7 months at 7% per annum?

Ans. $\$562.50 \times 1.25\frac{1}{12} = \$703.19+$.

7. To find the interest or amount of any sum for any number of years, months, and days, at any given per cent.

1st. Find the exact time in years, months, and days (if they are not given) by the rule given under Subtraction of Denominate Numbers. (Page 32.)

2d. Get the interest on \$1 at 6%; *first*, for the number of years; *second*, for the months, making $\frac{1}{2}$ cent or 5 mills for each month; *third*, 1 mill for every 6 days, and $\frac{1}{6}$ of a

mill for each day. Add these several results, which will be the 6% interest on $1, for the time.

3d. If any other than 5% is required, add to the amount at 6%, or subtract from it, such a part of this amount as the required per cent exceeds or falls short of 6%.

4th. Multiply the amount thus found by the principal, and the product will be the required interest. This interest added to the principal will give the amount.

43. What is the interest and the amount of $256.75, at 6% per annum for 3 years, 7 months, 11 days?

ILLUSTRATION. The interest of $1 for 3 years $= 3 \times 6\% = 18\%$; for 7 months $= \frac{7}{2}$ of a cent $= 3\frac{1}{2}$ cents, or $.035: and for 11 days $= \frac{11}{6}$ mills $= .001\frac{5}{6}$. These results added $= .18 + .035 + .001\frac{5}{6} = .216\frac{5}{6} \times \$256.75 = \$55.67$ = the interest $+ \$256.75 = \$312.42$.

44. What is the interest of $425.25 at 7% per annum, for 3 years, 8 months, 12 days?

ILLUSTRATION. 1st. The interest at 6%, as above shown, is $\$.18 + .04 + .002 = \$.222 + \frac{1}{6} \times .222 = .222 + .037 = \$.259 \times \$425.25 = \$110.14$ = *Ans.*

But if the rate were 4% in the last example, then the interest on $1 would be $\$222 - \frac{1}{3} \times 222 = .222 - .074 = .148 \times \$425.25 = \$62.937$.

45. What is the interest and amount of $568.75, on interest at 7% for 3 yrs, 11 mo., 29 days?

Ans. $159.12+.

8. Partial Payments.

Sometimes notes or bonds run on interest for a series of years, at a fixed per cent, when the payor makes several payments, which are usually indorsed on the note. The Supreme Court of the United States has established a rule by which the interest on such notes is cast.

1. *Find the amount of the given principal up to the time when the sum of the partial payments is equal to, or exceeds, the interest then due; from this result subtract the sum of the partial payments made within the time considered.*

2. *Take the remainder for a new principal, and proceed as before, continuing the operation to the time of settlement.*

46. *Common Note.*

$657.50. WASHINGTON, April 4, 1876.

For value received, I promise to pay Arthur McArthur or order, six hundred and fifty-seven dollars and fifty cents, on demand, with interest at 7 per cent.

W. W. CURTIS.

What was due on taking up the note, March 12, 1880?

The following payments were indorsed on the note:

Nov.	12, 1876,	$62.66
Aug.	14, 1877,	24.25
April	15, 1878,	20.75
July	20, 1879,	95.87

METHOD OF OPERATION.

					y.	m.	d.		
Find the time.	1880,	3d,	12th,	settled.				due	
	1879,	7,	20,	runs,		7,	22,	payment	$95.87
	1878,	4,	15,	"	1,	3,	5,	"	20.75
	1877,	8,	14,	"		8,	1,	"	24.25
	1876,	11,	12,	"		9,	2,	"	62.66
	1876,	4,	4,	date		7,	8,		

	m. d.	
	$657.50 ×	7, 8, @ 7% = $1.042\frac{7}{12}$.
	$1.042\frac{7}{12}$ =	
	$685.37	
Less	62.66 =	1st payment.
New prin.	$622.71 ×	The 2d and 3d payments not equal to interest.
Rate.	$1.183\frac{1}{3}$ =	int. on $1, and am't from Nov. 1876, to July 1879.
	736.92	less $24.25+$20.75+$95.87=three payments.
	140.87	
New prin.	596.05 ×	
	$1.045\frac{1}{3}$ =	amount of $1 from July 1879 to March, 1880.
	$623.0738	due March 12, 1880.

47. A note is dated Feb. 7, 1851, for $5642.35, at 6%. Indorsements: May 18, 1852, $748.75; Nov. 20, 1853, $652.33; March 6, 1854, $427.60; and Aug. 25, 1854, $74.50. How much was due Sept. 10, 1855?

Ans. $5181.83.

48. On a note dated June 15, 1874, for $1,500 at 7%, were these indorsements. Dec. 15, 1874, $300; May 30, 1875, $300; Dec. 18, 1875, $400. What was due July 15, 1877?
Ans. $700.452.

49. What was due July 9, 1877, on a note dated Jan. 6, 1875, for $1,280, at 7%, with the following indorsements? July 18, 1875, $175; Dec. 12, 1875, $375, and July 24, 1876, $400.
Ans. $474.887.

9. Examples under Stocks and Bonds.

The par value of stock is 100% of its real value. Stock is above par when $100 worth sells for more than $100; and below par when $100 worth sells for less than $100. In selling or buying stock, there is generally a certain per cent charged as brokerage; commonly ¼%. The *market value* is what the stock brings in the market. A share of stock is generally considered to be $100 and a bond $1000.

50. What is the market value of 120 shares of bank stock at $53\frac{3}{4}\%$. *Ans.* $\$12,000 \times .5375 = \$6,450$.

51. Sold 44 shares of bank stock at $129\frac{1}{2}\%$. What is the cost, including brokerage?

Ans. $129\frac{1}{2} + \frac{1}{4} = 1.2975 \times 4400 = \$5.709$.

52. A sold 160 shares of railroad stock at $92\frac{3}{4}\%$, and purchased with the proceeds bank stock at $73\frac{3}{4}\%$, paying brokerage on both sale and purchase; how many shares did he receive?

Ans. $\$16,000 \times .92\frac{1}{2} \div .74 \div 100 = 200$ shares.

53. What is the discount on 190 shares of R. R. stock, sold at 89%? *Ans.* \$2,090.

54. A broker sells 30 shares of bank stock at $96\frac{1}{2}\%$, and 120 shares of railroad stock at 105%, retaining the brokerage; how much does he pay to his principal?

Ans. \$15,457.50.

55. At what price must United States 4% bonds be purchased, to afford 5% on the investment?

Ans. $\frac{4}{5} \times 100 + 80\%$.

56. Paid \$6772.50 for securities bearing 8% interest. What was my income and the rate per cent of my investment, if purchased at a premium of $12\frac{7}{8}\%$?

Ans. $\$6,772.50 \div 1.12875 = \$6,000 \times .08 = \$480 =$ income, $\$480 \div \$6,772.50 = .07\frac{780}{903}\%$.

57. If I purchase 6% bonds at 106, and 5% at 90, which investment will yield the larger income?

ILLUSTRATION. $6\% \div 106\% = 5\frac{35}{53}\%$, and $5\% \div 90\% = 5\frac{5}{9}\%$. The difference between $5\frac{35}{53}\%$ and $5\frac{5}{9}\%$ is $\frac{53}{477}\%$, or $10\frac{239}{477}\%$ in favor of 6% bonds.

58. One year after buying steamboat stock at $97\frac{1}{2}\%$, I received an annual dividend of $9\frac{3}{4}\%$. What was the rate per cent of income realized on my investment?

Ans. $9\frac{3}{4}\% \div 97\frac{1}{2}\% = 10\%$.

59. Bought manufacturing stock at 10% discount, and received two semi-annual dividends of 4% each. My total income was \$648. What did I pay for the stock, and what rate of income on investment did the stock yield?

Ans. Rate of income, $8\frac{8}{9}$%; cost of stock, \$7290.

60. Dividends of 5% are annually declared on certain bank stock, and the par value of each share is \$100. A shareholder sold 17 shares at a price that would pay but 4% on investment. How much was paid for the stock?

Ans. $\frac{5}{4} \times \$1700 = \$2125$.

61. Bought 24 shares Boston & Albany R. R. stock at $79\frac{7}{8}$; received a semi-annual dividend of 4%, and then sold the stock for $88\frac{3}{4}$. How much did I gain?

Ans. \$309.

10. Examples under Exchange.

62. A merchant in Baltimore bought a quantity of wheat in Chicago, and remitted in payment a sight draft for \$1264, at $\frac{3}{4}$% discount. How much did the draft cost the merchant?

Ans. \$1254.52.

63. A firm in Louisville bought a 60 days' draft on New York for \$2500, at $\frac{3}{8}$% premium, interest 6%. What did the draft cost?

ILLUSTRATION. Find cost of \$1 – discount .0105 + premium .00375 = .99325 × 2500 = \$2483.125 = *Ans.*

64. A certain broker in New York bought a 90 days' 6% draft for \$1,200, on New Orleans, at $\frac{1}{2}$% discount. He paid $\frac{1}{16}$% additional for brokerage. How much did he pay for the draft?

Ans. \$1,176.15.

ILLUSTRATION. He pays for \$1, \$1 – the interest and discount = .0155 + .005 = .9795 + $\frac{1}{16}$% brokerage = $.0006\frac{1}{4}$ = $.9801\frac{1}{4}$ × 1200 = *Ans.*

65. An agent remitted to St. Louis from Detroit a 60 days' draft for \$480, which cost \$476.52. What was the rate of exchange, interest being 7% per annum?

Ans. $\frac{1}{2}$% premium.

ILLUSTRATION. The present worth of \$1 is \$.98775 × 480 = = \$474.12. Cost \$476.52 − 474.12 = \$2.40 ÷ \$480 = *Ans.*

66. I owe a sum to a merchant in Chicago, and if I remit my draft for payment from Philadelphia for 60 days at $\frac{7}{8}$% premium and 6% interest, it will cost \$1,197.90. What sum do I owe in Chicago? *Ans.* \$1,200.

ILLUSTRATION. \$1 − the interest \$.0105 + the premium, \$.00875 = \$.99825 = cost of \$1; then \$1,197.90 ÷ .99825 = \$1,200.

Additional test examples are given in the following miscellaneous test questions.

11. Examples in Equation of Payments, etc.

By the rules of *Equation of Payments*, we are to find the average time for the payment, in one sum, of several sums due at different dates, without loss of interest to either party.

Some date is generally assumed from which the time for each debt to run is found. When the time for each debt or account is found in months or days, or any fixed periods of time, we must find the value of *each sum* for *one* of the periods of time by multiplying the sum by the number of months, days, etc., it has to run, and the sum of the several products, divided by the sum of the payments, or of the accounts, will give the *equated number* of months, days, etc.

Or, the interest on each sum for the time it has to run may be found, and the sum of the interest on the several sums divided by the interest on the sum of the debts or accounts, for one month or day, or period of time, will give the equated or average time for the payment of all debts.

1. A merchant sold goods to the amount of \$3,600, $\frac{1}{2}$ of which was to be paid in 6 months, $\frac{1}{4}$ in 8 months, and the remainder in 9 months. What was the average term of credit? *Ans.* $7\frac{1}{4}$ months.

2. Bought an invoice of goods amounting to \$1,200, to be paid for as follows: \$600 in 30 days, \$300 in 60 days, and \$300 in 90 days. What is the equated time for the payment of the entire bill? *Ans.* $52\frac{1}{2}$ days.

3. If I purchase \$7,000 worth of merchandise, and pay $\frac{1}{7}$ in cash, and the balance, $\frac{1}{3}$ in 2 months, $\frac{1}{12}$ in 3 months, $\frac{1}{6}$ in 4 months, $\frac{1}{4}$ in 5 months, and the rest in 6 months, what is the equated time for paying the whole sum?

Ans. $3\frac{5}{6}$ months.

4. Find the equated time for the settlement of the following accounts:

Dr. WOODWARD & LATHROP, *in account with* W. A. COOK. *Cr.*

1886.			1866.		
Jan. 1,	To balance	\$283.94	Jan. 12,	By cash	\$425.00
May 15,	mdse.	217.33	April 3,	mdse.	693.13
July 5,	note	500.00	June 29,	mdse.	37.59
Aug. 30,	mdse.	894.60			

Ans. Equated time 355 days. Balance due 176 days before date found for Dr. side. *Ans.* Dec. 21, 1886.

ILLUSTRATION.

	Days.	Dr.		Days.	Cr.
\$283.94 ×	0 =				
217.33 ×	135 =	29.339.55	\$425.00 ×	12 =	\$5,100
500.00 ×	186 =	93.000.00	693.13 ×	93 =	64,461.09
894.60 ×	242 =	216.493.20	37.59 ×	180 =	6,766.20
1,895.87		338,832.75	1,155.72		76,327.29

\$338,832.75 ÷ 1,895.87 = 179 nearly = average date for Dr. side.
\$76,327.29 ÷ 1,155.72 = 66 nearly = average date for Cr. side.

5. I bought goods as follows: May 5, $500 on 4 months' credit; May 25, $750 on 4 months' credit; June 27, $800 on 6 months' credit. When shall I pay the whole debt?

Ans. Oct. 26.

ILLUSTRATION. Take May 5 for the initial date, due Sept. 5. May 25 is due Sept. 25, and June 27 is due Dec. 27.

$500 (initial)	× 0	=	.0000
750	× 20 days	=	15,000
800	× 113	=	90,400
$2,050			$105,400 ÷ 2050 = 51 days.

6. A owes B $8,750, payable July 21, 1877; and B owes A $6,500, payable June 9, 1877. When and to whom is the balance payable?

Ans. To B in 163 days from June 9, or Nov. 19, 1877.

7. A owes B $6,500, payable July 21, 1887, and B owes A $8,750, payable June 9, 1877. To whom is the balance payable and what is the equated time?

Ans. To A, and the time is 121 days, or Feb. 8, 1877.

8. Find the equated time of payment of the balance of the following account:

Dr. S. S. PACKARD *in account with* M. S. MATHER. *Cr.*

1886.			1886.		
April 1.	Merchandise	$375.00	April 20.	Cash,	$500.00
" 18.	"	250.00	May 20.	"	185.00
May 25.	"	150.00			

Equated time, Dr. side, 16 days from April 1 = April 17.
Equated time, Cr. side, 8 days from April 20 = April 28.

Again, April 17.	$775.00	April 28.	$685.00

Equated time 11 days. Balance = $90. The smaller due last. $685 × 11 = 7535 ÷ 90 = 84 days; subtracted from April 1, the date will be Jan. 23, 1886, for payment of balance.

Miscellaneous Test Questions.

1. A man has real estate worth \$20,114.50, bank stock worth \$15,779.82; United States bonds, worth \$17,772.89; and other property worth \$6,317.27. What is the value of his entire property?

2. A man, having a sum of money, earned \$8,211, and afterwards lost \$2,114, when he found he had \$11,415. How much had he at first?

3. A fox starts from a certain place, and runs at the rate of 616 yards in a minute; at the end of three minutes a dog starts after him, from the same place, and follows at the rate of 792 yards a minute; how far apart are they at the end of 9 minutes? *Ans.* 892 yards.

4. How many horses, worth \$112 apiece, can be bought for 28 oxen worth \$63 each, 52 cows worth \$42 each, 175 sheep worth \$6 each, and \$2,394 in cash?

Ans. 66 horses.

5. How many boxes of tea, each containing 24lbs., worth 75 cents a pound, must be given for 4 bins of wheat, each containing 145 bushels, worth \$1.80 per bushel?

6. What is the largest number that will exactly divide 441 and 567?

7. What are the prime factors of 408 and 740?

8. What is a common multiple of 3, 4, 8, 12?

9. What is the least common multiple of 4, 6, 9, 14, 16?

10. How many quarts are there in the smallest cask of cider that can be exactly measured by either a 3-quart, a 5-quart, or a 6-quart measure?

11. How many pounds of butter in 4 tubs weighing respectively $27\frac{1}{3}$lbs., $34\frac{3}{8}$lbs., $32\frac{4}{5}$lbs., and $29\frac{2}{5}$lbs.?

12. A laborer earned $\$18\frac{3}{8}$, and received $\$14\frac{1}{4}$ previously due him; he then bought a barrel of flour for \$12, and groceries to the amount of $\$8\frac{11}{12}$. How much money had he remaining? *Ans.* $\$11\frac{17}{24}$.

13. What will $\frac{3}{5}$ of $\frac{5}{8}$ of a yard of cloth cost at the rate of $\frac{8}{17}$ of $\$3\frac{2}{5}$ per yard? *Ans.* $\$\frac{3}{5}$, or 60 cents.

14. A bought $\frac{7}{8}$ of a ship, and sold $\frac{3}{5}$ of his share to B. B then sold $\frac{2}{7}$ of what he bought to C for \$3,000. What was the whole ship worth at that rate? *Ans.* \$20,000.

15. Find by the shortest method the product of 28,452 feet by $33\frac{1}{3}$.

16. In five piles of wood there are respectively 4,316 cords; 8.23, 11.25, 7.364, and 13.819 cords. How many cords in all the piles? *Ans.* 44.979 cords.

17. From a hogshead of sugar containing 397.25lbs., a grocer sold parcels as follows: 110.25lbs., 64.5lbs., 14.25 lbs., 29.375lbs., 39.23lbs., and 16.33lbs. How much was left?

18. A man made a journey as follows: he traveled $7\frac{3}{4}$ hours by rail, at the rate of 22.75 miles an hour; $9\frac{7}{8}$ hours by stage, at the rate of 6.75 miles an hour; and 11.75 hours on foot, at the rate of 4.62 miles an hour. What was the length of the journey? *Ans.* 297.254 miles.

19. What is the sum of the quotients of $24 \div 96$, $42.75 \div 11.4$, and of $17.85 \div 4.2$? *Ans.* 8.25.

20. A tailor cuts from 31.25 yards of cloth, 6 coats, each taking 3.75 yards, and makes the rest into vests, each taking 1.25 yards. How many vests does he make? *Ans.* 7 vests.

21. When A was 14 years, 6 months old, he went out to service, and lived at one place 1 year, 9 months; at an-

other 2 years, 5 months; and at a third 3 years, 9 months. How old was he finally?

22. The longitude of New York is 74°, 3′, and that of San Francisco 122°, 26′, 45″. If it is 5 P. M. at New York, what time is it at San Francisco?

Ans. 1h., 46mi., 25sec., P. M.

23. A rectangular block of stone is 6ft. long, 3ft. wide, $2\frac{1}{2}$ft. thick. What is its weight if each cubic foot weighs 156lbs.? *Ans.* 7,026lbs.

24. A room is 24ft. long, $19\frac{1}{2}$ft. wide, $10\frac{1}{2}$ ft. high. What will it cost to plaster the sides and ceiling at 30 cents a square yard, after deducting $\frac{1}{5}$ for doors and windows?

Ans. $36.84.

25. The circumference of the fore wheel of a carriage is 13ft., 9in., and that of the hind wheel, 16ft., 6in. How many more times will the fore wheel go around than the hind wheel, in a journey of 30 miles? *Ans.* 1,920.

26. In a journey of 1,664 miles, A travels 208 miles by stage, and the rest of the way by rail. What per cent of the journey does he travel by rail? *Ans.* $87\frac{1}{2}$%.

27. Bought the following articles: $27\frac{1}{2}$ meters of linen, at 3.50 francs per meter; $6\frac{1}{4}$ meters of velvet at 17 francs per meter; 29 meters of brocade at 16.75 francs per meter; $28\frac{3}{4}$ meters of merino at 4 francs per meter; $1\frac{1}{2}$ dozen pairs of socks at 3.05 francs per pair; and 7 pairs of gloves at 42 francs a dozen pairs. What was the whole cost?

Ans. 882.65 francs.

28. A man's salary is $4,200. Of this he spends 22% for fuel and rent; 15% for clothing; $1,218 for other purposes. What per cent of his salary does he save? *Ans.* 34%.

29. A man bought 75 acres of land at 42\frac{3}{5}$ per acre, and sold it all for $3,577$\frac{1}{2}$. What per cent did he gain?

Ans. 12$\frac{1}{2}$%.

30. A broker receives $3,500 to buy cotton at 8 cents a pound, his commission being 1$\frac{3}{4}$%. How much does he buy?

Ans. 42,997$\frac{1}{2}$lbs.

31. A man bought a house for $3,224.24, including commissions at 2$\frac{1}{2}$%. What was the price of the house?

Ans. $3,145.60.

32. A commission merchant sold goods for $7,500, at 3$\frac{1}{2}$% commission. What did he make?

Ans. $262.50.

33. I insure my house for $8,000, for 3 years, at $\frac{1}{4}$% per annum. What is the total premium? *Ans.* $60.

34. A grocer bought 500 bags of coffee, each bag containing 49$\frac{1}{4}$lbs., at 12cts. per pound, and sold it at a profit of 16$\frac{2}{3}$%. For how much did he sell it? *Ans.* $3,447.50.

35. A man sold a pair of horses for $224 and gained 40%. What was their cost? *Ans.* $160.

36. A flock of sheep increases from 88 to 110 in a year. What is its per cent of increase? *Ans.* 25%.

37. A town is to be taxed $13,848. Its assessed valuation is $1,452,000, and it contains 390 polls, each liable to a tax of $2. Now if A is liable for 2 polls and an assessed valuation of $7,820, what is his whole tax?

Ans. $74.38.

38. What is the interest of $1,766, for 1 year, 1 month, 18 days, at 6%?

39. Find the amount of $5,000 on interest for 1 year, 7 months, 11 days, at 7%. *Ans.* $5,564.861.

40. What is the interest of \$124.20 at 7% for 63 days, reckoning 360 days for a year? *Ans.* \$2,965.

41. What is the exact interest on \$1,815, for 93 days at 5%? *Ans.* \$23,095.

42. The interest on \$450 for 3 years, 6 months, 18 days, is \$79.87$\frac{1}{2}$. What is the rate per annum? *Ans.* 5%.

43. How long will it take \$3,000, at 5% per annum, to yield \$600 interest? *Ans.* 4 years.

44. What sum of money will yield \$7.66$\frac{2}{3}$ interest, in 15$\frac{1}{3}$ months, at 6%? *Ans.* \$100.

45. In what time will \$5,000 at 7% produce the same interest as \$9,625, at 6$\frac{1}{2}$%, in 4 years, 5 months, 18 days? *Ans.* 7 years, 11 months, 24 days.

46. On a note dated Sept. 18, 1873, for \$7,000, at 6%, were these indorsements: July 6, 1874, \$500; Sept. 24, 1875, \$1,500; and Dec. 6, 1875, \$1,000. What was due July 12, 1876? *Ans.* \$5,081.63.

47. What is the amount of \$1,350 for 5 years, 4 months, at 6% interest compounded semi-annually? *Ans.* \$1,850.55.

48. Sold 500 bushels of oats at 62$\frac{1}{2}$ cents a bushel, 5% off, for cash. What was the cash price per bushel? *Ans.* 59$\frac{3}{8}$ cents.

49. What is the present value of \$10,000, due in 4 months, 18 days, at 4$\frac{1}{2}$%? *Ans.* \$9,830.425.

50. A note at bank for \$1,620, dated July 7, 1877, and payable 90 days after date, is discounted July 25, 1877, at 8%. What are the proceeds? *Ans.* \$1,593.

51. Find the proceeds of a note payable 30 days after date, at 10% per annum, that will yield $1,000.

Ans. (Proceeds of $1 = $.991⅔) = 1,000 ÷ .991⅔.

52. A broker sells 30 shares of bank stock at 96½%, and 120 shares of railroad stock at 105%, retaining the brokerage (¼%). How much does he pay to his principal?

ILLUSTRATION. 30 + 120 = 150 × $100 × 99½% = $15.495 less brokerage $37.50 = $15,457.50. *Ans.*

53. At what rate must an 8% stock be purchased to yield the purchaser 7% interest? *Ans.* 114 2/7%.

54. What is the cost of a sight draft on New York, for $1,250, exchange being 1¼% discount? *Ans.* $1,234.37½.

55. Find the cost of a sight draft on New York, for $2,400, payable 90 days after sight, interest being 10%, and exchange 103%.

Ans. $2,400 + difference of interest and premium = $2,410.

56. Find the market value of sight draft on New York, for $1,800, exchange being 99%. *Ans.* $1,782.

57. Find the value of a draft on New York, for $1,650, payable 60 days after sight, exchange being 98½%, and interest 6%.

ILLUSTRATION. $1,650 × 98½% less interest of $1 for 63 days = 1,650 × .9745 = $1,607.92½. *Ans.*

58. What is the face of a draft at 30 days, which costs $2,000, exchange being 102%, and interest 6%?

ILLUSTRATION. Exchange less interest = value of $1 = $1.0145; then $2,000 ÷ 1.0145 = $1,971.41½. *Ans.*

59. Find the cost of a 60 day bill on London, for £315, when £1 at 60 days is worth $480, gold being quoted at 105.

ILLUSTRATION. £315 × $4.80 = $1,512 × 105 = $1,587.60. *Ans.*

60. How large a bill on London can be bought for \$2,000 in currency, when sterling exchange is quoted at \$4.85 and gold at 1[illegible]?

Ans. \$2,000 ÷ 4.85 × 1[illegible] = £[illegible] 3s. 1d.

61. How large a bill on Paris can be bought for \$1,000 in gold, when exchange is at a rate of \$1 for 5.20 francs?

Illustration. 5.20 × 1000 = 5200 francs. *Ans.*

62. A merchant owes \$300, payable as follows: \$80 in 22 days, \$100 in 60 days, and \$120 in 75 days. What is the equated time for the payment of the whole?

Ans. $55\frac{13}{15}$ days.

63. If 24 yards, 3 quarters, of carpeting, 1 yard wide, will cover a certain room, how many yards $1\frac{3}{4}$ yards wide will cover the same room?

Ans. $1\frac{3}{4} : 1 :: 24\frac{3}{4}$: *Ans.*

64. If a man can do a piece of work in 20 days, working 10 hours a day, how long will it take him to do the same work if he works 12 hours a day?

65. If a man can walk 250 miles in 9 days of 12 hours each, how many days of 10 hours each would it take him to walk 400 miles?

66. If 2 bushels, 1 peck of wheat cost \$2.43, what will $14\frac{1}{2}$ bushels cost? *Ans.* \$15.66.

67. A, B, and C enter into speculation: A puts in \$4,000, B puts in \$5,000, and C puts in \$6,000; they lose \$2,000. What part must each lose?

THE END.

GRUBE'S METHOD

OF TEACHING ARITHMETIC EXPLAINED. WITH A LARGE NUMBER OF PRACTICAL HINTS AND ILLUSTRATIONS.

BY PROF. F. LOUIS SOLDAN,

Principal of the Normal School, St. Louis, Mo.

Many teachers know of "Grube's Method" by the hearing of the ear. They will be thankful for the day which first gave them a clear understanding of it. Here it is in full, by one of the most eminent and successful teachers in the country. The first and only publication of the kind in this country. Price 30 cents.

OUTLINES OF MAP-DRAWING,

WITH DIAGRAMS FOUNDED ON PARALLELS AND MERIDIANS.

BY F. E. BANGS,

PRINCIPAL OF WOOSTER SCHOOL, NEW HAVEN, CONN.

FROM THE INTRODUCTION.

"The aim of the author has been to construct approximately accurate outline-maps by the use of diagrams founded on parallels and meridians. These lines are common to all maps, and the system here presented may be used in connection with any series of geographies.

"The simplicity and uniformity of the system are its merits. The rectangle, 5 by 4 inches; is common to all the maps. The side lines on the sectional maps of the United States, and the centre meridian on the maps of the continents, are divided into equal parts; and the length of one degree, as measured on these lines, is taken as the unit of measurement. The relative areas of the sections, or countries, may be found by comparing the squares, each square being one square degree, or equal to the area of the State of Connecticut.

"In the sectional maps of the United States the side lines are divided into a given number of equal parts. The slant of the meridian is determined by a convenient measurement. The remainder of the top line, and the base line, are divided into the same number of equal parts as the side line, giving the location of the parallels and meridians. These sectional maps, with their diagrams, are readily combined to form the map of the United States.

"On the maps of the continents, the centre meridian is divided into equal parts, giving the points through which any parallels may be drawn, and determining the position of the tropics and polar circles. The parallels of North America, Europe, and Asia, are formed by circles whose centre is at a given point above the top line, — the north pole. The parallels of the other continents are formed by straight lines.

"The hemispheres are presented in outline, and are easily completed. These outlines enclose certain prominent geographical features, which determine the position and direction of the remainder. The parallels and meridians are formed by circles with different centres.

"The list of topics is designed to accompany the drawing of the map, and to present in the outline the prominent physical features of the section or country. The cities are grouped, by states or countries, in order of size and with reference to their leading industries. The description of these countries and cities, as indicated in the list of topics, may be found in the text-book and gazetteer.

"This system is a product of the schoolroom, and has been taught successfully, both in the primary and intermediate grades, proving itself fully within the ability of young pupils to comprehend and practice."

The maps are printed on tinted plates ruled to show comparative area; the diagrams and directions are simple and definite; and the list of topics, while a guide to the drawing, furnishes valuable suggestions and information. The work contains suggestions to teachers, a relief-map for each of the continents, with directions for molding. The system is complete in itself, and may be used with any text-book on geography. Price 30 cents.

THE INTERSTATE PUBLISHING COMPANY

CHICAGO AND BOSTON